

# **CZ: Multimethods and Multiple Inheritance Without Diamonds<sup>1</sup>**

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## **Abstract**

Multiple inheritance has long been plagued with the “diamond” inheritance problem, leading to solutions that restrict expressiveness, such as mixins and traits. Instead, we address the diamond problem directly, considering two difficulties it causes: ensuring a correct semantics for object initializers, and typechecking multiple dispatch in a modular fashion—the latter problem arising even with multiple interface inheritance. We show that previous solutions to these problems are either unsatisfactory or cumbersome, and suggest a novel approach: supporting multiple inheritance but forbidding diamond inheritance. Expressiveness is retained through two features: a “requires” construct that provides a form of subtyping without inheritance (inspired by Scala [39]), and a dynamically-dispatched “super” call similar to that found in traits. Through examples, we illustrate that inheritance diamonds can be eliminated via a combination of “requires” and ordinary inheritance. We provide a sound formal model for our language and demonstrate its modularity and expressiveness.



# 1 Introduction

Single inheritance, mixins [12, 5, 24, 22, 4, 9], and traits [19, 23, 39] each have disadvantages: single inheritance restricts expressiveness, mixins must be linearly applied, and traits do not allow state. Multiple inheritance is one solution to these problems, as it allows code to be reused along multiple dimensions. Unfortunately however, multiple inheritance poses challenges itself.

There are two types of problems with multiple inheritance: (a) a class can inherit multiple features with the same name, and (b) a class can have more than one path to a given ancestor (i.e., the “diamond problem”, also known as “fork-join” inheritance) [42, 45]. The first, the conflicting-features problem, can be solved by allowing renaming (e.g., Eiffel [31]) or by linearizing the class hierarchy [46, 45]. However, there is still no satisfactory solution to the diamond problem.

The diamond problem arises when a class  $C$  inherits an ancestor  $A$  through more than one path. This is particularly problematic when  $A$  has fields—should  $C$  inherit multiple copies of the fields or just one? Virtual inheritance in C++ is designed as one solution for  $C$  to inherit only one copy of  $A$ ’s fields [21]. But with only one copy of  $A$ ’s fields, object initializers are a problem: if  $C$  transitively calls  $A$ ’s initializer, how can we ensure that it is called only once? Existing solutions either restrict the form of constructor definitions, or ignore some constructor calls [38, 21].

There is another consequence of the diamond problem: it causes multiple inheritance to interact poorly with modular typechecking of multiple dispatch. Multiple dispatch is a very powerful language mechanism that provides direct support for extensibility and software evolution [14, 16]; for these reasons, it has been adopted by designers of new programming languages, such as Fortress [2]. Unfortunately however, modular multimethods are difficult to combine with any form of multiple inheritance—even restricted forms, such as traits or Java-style multiple interface inheritance. Previous work either disallows multiple inheritance across module boundaries, or burdens programmers by requiring that they always provide (possibly numerous) disambiguating methods.

To solve these problems, we take a different approach: while permitting multiple inheritance, we disallow inheritance diamonds entirely. So that there is no loss of expressiveness, we divide the notion of inheritance into two concepts: an *inheritance dependency* (expressed using a *requires* clause, an extension of a Scala construct [38]) and implementation inheritance (expressed through *extends*). Through examples, we illustrate that programs that are expressed using diamond inheritance can be translated to a hierarchy that uses a combination of *requires* and *extends*, without the presence of diamonds. As a result, our language, CZ—for cubic zirconia—retains the expressiveness of diamond inheritance.

We argue that a hierarchy with multiple inheritance is conceptually two or more separate hierarchies. These hierarchies represent different “dimensions” of the class that is multiply inherited. We express dependencies between these dimensions using *requires*, and give an extended example of its use in Sect. 5.

Our solution has two advantages: fields and multiple inheritance (including initializers) can gracefully co-exist, and multiple dispatch and multiple inheritance can be combined. To

achieve the latter, we make an incremental extension to existing techniques for modular type-checking of multiple dispatch.<sup>1</sup>

An additional feature of our language is a dynamically-dispatched super call, modelled after trait super calls [19]. When a call is made to  $A.\text{super}.f()$  on an object with dynamic type  $D$ , the call proceeds to  $f$  defined within  $D$ 's immediate superclass along the  $A$  path. With dynamically-dispatched super calls and `requires`, our language attains the expressiveness of traits while still allowing classes to inherit state.

We have formalized our system as an extension of Featherweight Java (FJ) [28] (Sect. 8) and have proved it sound (Appendix B).

### Contributions:

- The design of a novel multiple inheritance scheme<sup>2</sup> that solves (1) the object initialization problem and (2) the modular typechecking of multimethods, by forbidding diamond inheritance (Sections 2 and 4).
- Generalization of the `requires` construct and integration with dynamically-dispatched super calls (Sect. 6).
- Examples that illustrate how a diamond inheritance scheme can be converted to one without diamonds (Sections 3 and 5).
- Examples from actual C++ and Java programs, illustrating the utility of multiple inheritance and inheritance diamonds (Sect. 7).
- A formalization of the language, detailed argument of modularity (Sect. 8), and proof of type safety.
- An implementation of a typechecker for the language, as an extension of the JastAddJ Java compiler [20].

## 2 Object Initialization

To start with, diamond inheritance raises a question: should class  $C$  with a repeated ancestor  $A$  have two copies of  $A$ 's instance variables or just one—i.e., should inheritance be “tree inheritance” or “graph inheritance” [13]? As the former may be modelled using composition, the latter is the desirable semantics; it is supported in languages such as Scala, Eiffel, and C++ (the last through virtual inheritance) [38, 31, 21]. Unfortunately, the object initialization problem occurs in this semantics, depending how and when the superclass constructor or initializer is called [46, 45].

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<sup>1</sup>For simplicity and ease of understanding, our formal system only allows dispatch on a method's receiver and its first argument, corresponding to double dispatch. The system can be easily generalized to  $n$ -argument multimethods, as all interesting typechecking issues also arise in the two-argument case. See Sect. 8 for further details.

<sup>2</sup>This is a revised and expanded version of a paper presented at the FOOL '09 workshop [30]. (FOOL has no archival proceedings and is not intended to preclude later publication.) The main changes are the inclusion of multimethods rather than external methods and a new section on real-world examples (Sect 7).

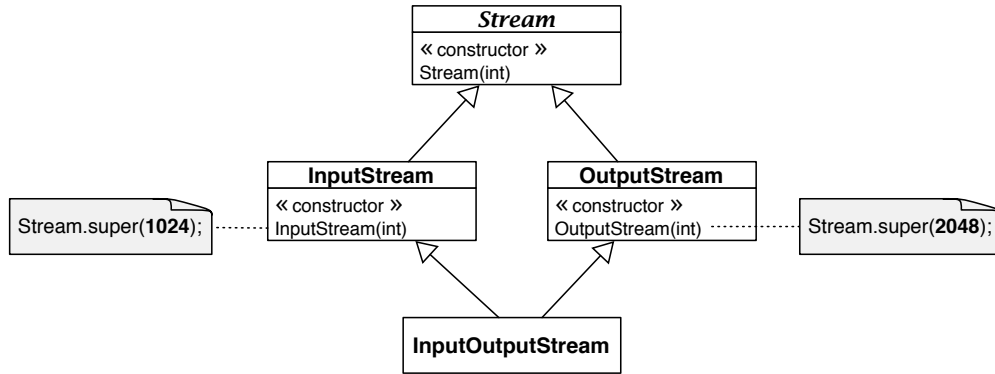


Figure 1: An inheritance diamond. Italicized class names indicate abstract classes.

**The Problem.** To illustrate the problem, consider Figure 1, which shows a class hierarchy containing a diamond. Suppose that the *Stream* superclass has a constructor taking an integer, to set the size of a buffer. *InputStream* and *OutputStream* call this constructor with different values (1024 and 2048, respectively). But, when creating an *InputOutputStream*, with which value should the *Stream* constructor be called? Moreover, *InputStream* and *OutputStream* could even call different constructors with differing parameter types, making the situation even more uncertain.

**Previous Solutions.** Languages that directly attempt to solve the object initialization problem include Eiffel [31], C++ [21], Scala [38] and Smalltalk with stateful traits [8].

In Eiffel, even though (by default) only one instance of the repeatedly inherited class is included (e.g., *Stream*), when constructing an *InputOutputStream*, the *Stream* constructor is called twice. This has the advantage of simplicity, but unfortunately it does not provide the proper semantics; *Stream*'s constructor may perform a stateful operation (e.g., allocating a buffer), and this operation would occur twice.

In C++, if virtual inheritance is used (so that there is only one copy of *Stream*), the constructor problem is solved as follows: the calls to the *Stream* constructor from *InputStream* and *OutputStream* are ignored, and *InputOutputStream* must call the *Stream* constructor explicitly.<sup>3</sup> Though the *Stream* constructor is called only once, this awkward design has the problem that constructor calls are ignored. The semantics of *InputStream* may require that a particular *Stream* constructor be called, but the language semantics would ignore this dependency by bypassing the constructor call.

Scala provides a different solution: trait constructors may not take arguments. (Scala traits are abstract classes that may contain state and may be multiply inherited.) This ensures that *InputStream* and *OutputStream* call the same super-trait constructor, causing no ambiguity for *InputOutputStream*. Though this design is simple and elegant, it restricts expressiveness (in fact, the Scala team is currently seeking a workaround to this problem [49]).

Smalltalk with stateful traits [8] does not contain constructors, but by convention, objects

<sup>3</sup>Since there is no default *Stream* constructor, this call cannot be automatically generated.

are initialized using an initialize message. Unfortunately, this results in the same semantics as Eiffel; here, the Stream constructor would be called twice [7]. The only way to avoid this problem would be to always define a special initializer that does not call the superclass initializer. Requiring that the programmer define such a method essentially means that the C++ solution must be hand-coded. Aside from being tedious and error-prone, this has the same drawbacks as the C++ semantics.

Mixins and traits do not address the object initialization problem directly, but instead restrict the language so that the problem does not arise in the first place. We compare CZ to each of these designs in Sect. 3.2.

## 3 An Overview of CZ

This section describes the CZ language design at a high level, including a description of how CZ addresses the object initialization problem and a comparison to related language designs.

### 3.1 CZ Design

CZ's design is based on the intuition that there are relationships between classes that are not captured by inheritance, and that if class hierarchies could express richer interconnections, inheritance diamonds need not exist. Suppose the concrete class  $C$  extends  $A$ . As noted by Schärli et al., it is beneficial to recognize that  $C$  serves two roles: (1) it is a generator of instances, and (2) it is a unit of reuse (through subclassing) [43]. In the first role, inheritance is the implementation strategy and may not be omitted. In the second role, however, it is possible to transform the class hierarchy to one where an inheritance *dependency* between  $C$  and  $A$  is stated and where *subclasses* of  $C$  inherit from both  $C$  and  $A$ . The key distinguishing feature of CZ is this notion of inheritance dependency, because while multiple inheritance is permitted, inheritance diamonds are forbidden.

Consider the inheritance diamond of Fig. 1. To translate this hierarchy to CZ, `InputStream` would be made abstract and its relationship to `Stream` would be changed from inheritance to an inheritance *dependency*, requiring that (concrete) subclasses of `InputStream` also inherit from `Stream`. In other words, `InputStream` *requires the presence of Stream in the extends clause of concrete subclasses*, but it need not extend `Stream` itself. Since `InputStream` is now abstract (making it serve only as a unit of reuse), it can be safely treated as a subtype of `Stream`. However, any concrete subclasses of `InputStream` (generators of instances), must also inherit from `Stream`. Accordingly, `InputOutputStream` must inherit from `Stream` directly.

We have reified this notion of an inheritance dependency using the `requires` keyword, a generalized form of a similar construct in Scala [39, 38].<sup>4</sup>

**Definition 3.1** (Subclassing). The subclass relation is defined as the reflexive, transitive closure of the `extends` relationship.

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<sup>4</sup>In Scala, `requires` is used to specify the type of a method's receiver (i.e., it is a selftype), and does not create a subtype relationship. As far as the Scala team is aware, our proposed use of `requires` is novel [49].



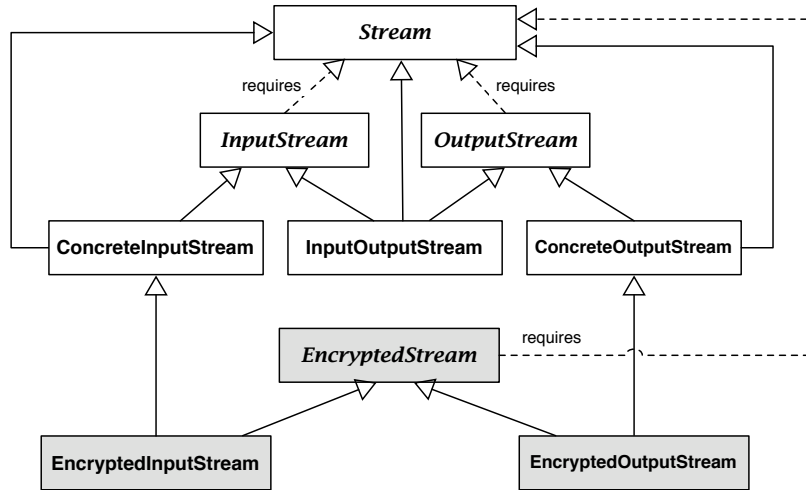


Figure 2: The stream hierarchy of Fig. 1, translated to CZ, with an encryption extension in gray. Italicized class names indicate abstract classes, solid lines indicate extends, and dashed lines indicate requires.

**Definition 3.2** (Requires).

When a class  $C$  requires a class  $B$ , we have the following:

- $C$  is abstract
- $C$  is a subtype of  $B$  (but not a subclass)
- Subclasses of  $C$  must either require  $B$  themselves (making them abstract) or extend  $B$  (allowing them to be concrete). This is achieved by including a `requires  $B'$`  or `extends  $B'$`  clause, where  $B'$  is a subclass of  $B$ .

In essence,  $C$  requires  $B$  is a contract that  $C$ 's concrete subclasses will extend  $B$ .

The revised stream hierarchy is displayed in Fig. 2. In the original hierarchy, `InputStream` served as both generator of instances and a unit of reuse. In the revised hierarchy, we divide the class in two—one for each role. The class `ConcreteInputStream` is the generator of instances, and the abstract class `InputStream` is the unit of reuse. Accordingly, `InputStream` requires `Stream`, and `ConcreteInputStream` extends both `InputStream` and `Stream`. The concrete class `InputOutputStream` extends each of `Stream`, `InputStream`, and `OutputStream`, creating a *subtyping* diamond, but not a *subclassing* diamond, as `requires` does not create a subclass relationship.

The code for `InputStream` will be essentially the same as before, except for the call to its super constructor (explained further below). Because `InputStream` is a subtype of `Stream`, it may use all the fields and methods of `Stream`, without having to define them itself.

Programmers may add another dimension of stream behavior through additional abstract classes, for instance `EncryptedStream`. `EncryptedStream` is a type of stream, but it need not extend `Stream`, merely require it. Concrete subclasses, such as `EncryptedInputStream` must inherit

from `Stream`, which is achieved by extending `ConcreteInputStream`. (It would also be possible to extend `Stream` and `InputStream` directly.)

The `requires` relationship can also be viewed as declaring a semantic “mixin”—if  $B$  requires  $A$ , then  $B$  is effectively stating that it is an extension of  $A$  that can be “mixed-in” to clients. For example, `EncryptedStream` is enhancing `Stream` by adding encryption. Because the relationship is explicitly stated, it allows  $B$  to be substitutable for  $A$ .

Using `requires` is preferable to using `extends` because the two classes are more loosely coupled. For example, we could modify `EncryptedInputStream` to require `InputStream` (rather than extend `ConcreteInputStream`). A concrete subclass of `EncryptedInputStream` could then also extend a *subclass* of `InputStream`, such as `BufferedInputStream`, rather than extending `InputStream` directly. In this way, different pieces of functionality can be combined in a flexible manner while avoiding the complexity introduced by inheritance diamonds.

**Object initialization.** Because there are no inheritance diamonds, the object initialization problem is trivially solved. Note that if class  $C$  requires  $A$ , it need not (and should not) call  $A$ 's constructor, since  $C$  does not inherit from  $A$ . In our example, `InputStream` does not call the `Stream` constructor, while `ConcreteInputStream` calls the constructors of its superclasses, `InputStream` and `Stream`. Thus, a *subtyping* diamond does not cause problems for object initialization.

This may seem similar to the C++ solution; after all, in both designs, `InputStream` calls the `Stream` constructor. However, the CZ design is preferable for two reasons: a) there are no constructor calls to non-direct superclasses, and, more importantly, b) no constructor calls are ignored. In the C++ solution, `InputStream` may expect a particular `Stream` constructor to be called; as a result, it may not be properly initialized when this call is ignored. Essentially, CZ does not allow the programmer to create constructor dependencies that cannot be enforced.

**Using “requires”.** Introducing two kinds of class relationships raises the question: when should programmers use `requires`, rather than `extends`? A rule of thumb is that `requires` should be used when a class is an extension of another class and is itself a unit of reuse. If necessary, a concrete class extending the required class (such as `ConcreteInputStream`) could also be defined to allow object creation. Note that this concrete class definition would be trivial, likely containing only a constructor. On the other hand, when a class hierarchy contains multiple disjoint alternatives (such as in the AST example in the next section), `extends` should be used; the no-diamond property is also a semantic property of the class hierarchy in question.

The above guideline may result in programmers defining more abstract classes (and corresponding concrete classes) than they may have otherwise used. However, some argue that it is good design to make a class abstract whenever it can be a base class. This is in accordance with the design of classes in Sather [48], traits in Scala and Fortress [38, 2, 3], and the advice that “non-leaf” classes in C++ be abstract [32]. In Sather and Fortress, for example, only abstract classes may have descendants; concrete classes (called “objects” in Fortress) form the leaves of the inheritance hierarchy [48]. Furthermore, a language could define syntactic sugar to ease the task of creating concrete class definitions; we sketch such a design in Sect. 6.4.

## 3.2 Related Work

**Subtyping and subclassing.** Since `requires` provides subtyping without subclassing, our design may seem to bear similarity to other work that has also separated these two concepts (e.g. [27, 18, 48, 15, 29]). There is an important difference, however, regarding information hiding. In a language that separates subclassing and subtyping, an “interface” type cannot contain private members; otherwise superclasses would be able to access private members defined in subclasses. Unfortunately, this restriction can be problematic for defining binary methods such as the `equals` method; its argument type must contain those private members for the method be able to access them. But, for this type to contain private members, it must be tied to a particular class implementation, as only subclasses (as opposed to subtypes) should conform to this type. See Appendix A for an example illustrating this issue.

This difficulty does not arise when using `requires`, as it establishes a stronger relationship than just subtyping; concrete subclasses must (directly or indirectly) inherit from the required class, as opposed to any class that provides a particular interface. Therefore, `Stream` may define an `equals(Stream)` method, and objects of type e.g. `InputStream`, `OutputStream`, or `InputStream` may be safely passed to this method. Since the private member is defined in `Stream` and is only accessed by a method of `Stream`, this does not violate information hiding.

**Mixins.** Mixins, also known as abstract subclasses, provide many of the reuse benefits of multiple inheritance while fitting into a single inheritance framework [12, 5, 24, 22, 4, 9]. While mixins allow defining state, they have the drawbacks that they must be explicitly linearized by the programmer and they cannot inherit from one another (though most systems allow expressing implementation dependencies, such as abstract members). If mixin inheritance were allowed, this would be essentially equivalent to Scala traits, which *do* have the object initialization problem. Additionally, the lack of inheritance has the consequence that mixins do not integrate well with multiple dispatch; multiple dispatch requires an explicit inheritance hierarchy on which to perform the dispatch.

**Traits.** Traits were proposed as a mechanism for finer-grained reuse, to solve the reuse problems caused by mixins and multiple inheritance [19, 23, 39]. In particular, the linearization imposed by mixins can necessitate the definition of numerous “glue” methods [19]. This design avoids many problems caused by multiple inheritance since fields may not be defined in traits.

Unfortunately, this restriction results in other problems. In particular, non-private accessors in a trait negatively impact information hiding: if a trait needs to use state, this is encoded using abstract accessor methods, which must then be implemented by the class composed using the trait. Consequently, it is impossible to define “state” that is private to a trait—by definition, all classes reusing the trait can access this state. Additionally, introducing new accessors in a trait results in a ripple effect, as all client classes must now provide implementations for these methods [8], even if there are no other changes.

In contrast, CZ allows a class to multiply inherit other classes, which may contain state. In particular, a class may extend other concrete classes, while in trait systems, only traits may be multiply inherited.

**Stateful traits.** Stateful traits [8] were designed to address the aforementioned problems with stateless traits. But, as previously mentioned, this language does not address the problem of a correct semantics for object initialization in the presence of diamonds. Additionally, stateful traits do not address the information hiding problem, as they have been designed for maximal code reuse. In this design, state is hidden by default, but clients can “unhide” it, and may have to resort to merging variables that are inherited from multiple traits. While this provides a great deal of flexibility for trait clients, this design does not allow traits to define private state.

## 4 Modular Multiple Dispatch

CZ also supports multiple dispatch, which we and others believe is more natural and more expressive than single dispatch [14, 16, 15]. In fact, one common source of bugs in Java programs occurs when programmers expect static overloading to behave in a dynamic manner [26]. Multiple dispatch also avoids the extensibility problem inherent in the Visitor pattern, as well as the complexity introduced by manual double dispatch. However, typechecking multiple dispatch in a modular fashion is very difficult in the presence of *any* form of multiple inheritance—precisely because of the diamond problem.

### 4.1 The Problem

To see why diamond inheritance causes problems, suppose we have the original diamond stream hierarchy, and we now define a multimethod `seek` in a helper class (supposing that such functionality did not already exist in the classes in question):

```
class StreamHelper {
    void seek(Stream s, long pos) {
        // default implementation: do nothing
    }
    void seek(Stream@InputStream is, long pos) {
        // seek if pos <= eofPos
    }
    void seek(Stream@OutputStream os, long pos) {
        // if pos > eofPos, fill with zeros
    }
}
```

The declaration `seek(Stream@InputStream, long)` specifies that the method *specializes* `seek(Stream, long)` for the case that the first argument *dynamically* has type `InputStream`.

Unfortunately, in the context of our diamond hierarchy, this method definition is ambiguous—what if we perform the call `h.seek(new InputOutputStream(), 1024)`? Unfortunately, it is difficult to perform a *modular* check to determine this fact. When typechecking the definition of `seek()`, we cannot search for a potential subclass of both `InputStream` and `OutputStream`, as this analysis would not be modular. And, when typechecking `InputOutputStream`, we cannot search for multimethods defined on both of its superclasses,

as that check would not be modular, either. We provide a detailed description of the conditions for modularity in Sect. 8.1.

It is important to note that this problem is *not* confined to multiple (implementation) inheritance—it arises in any scenario where an object can have multiple dynamic types on which dispatch is performed. For instance, the problem appears if dispatch is permitted on Java interfaces, as in JPred [25], or on traits, as in Fortress [3, 2]. For this reason, some languages restrict the form of dispatch to the single-inheritance case; e.g., MultiJava disallows dispatching on interfaces [16, 17].

## 4.2 Previous Solutions

There are two main solutions to the problem of modular typechecking of multiple dispatch in the presence of multiple inheritance. The first solution is simply to restrict expressiveness and disallow multiple inheritance across module boundaries; this is the approach taken by the “System M” type system for Dubious [36].

JPred [25] and Fortress [3] take a different approach. The diamond problem arises in these languages due to multiple interface inheritance and multiple trait inheritance, respectively. In these languages, the typechecker ensures that multimethods are unambiguous by requiring that the programmer always specify a method for the case that an object is a subtype of two or more incomparable interfaces (or traits). In our streams example, the programmer would have to provide a method like the following in the StreamHelper class (in JPred syntax):

```
void seek(Stream s, long pos) when s@InputStream && s@OutputStream
```

(In Fortress, the method would be specified using intersection types.) Note that in both languages, this method would have to be defined for *every* subset of incomparable types (that contains at least 2 members), regardless of whether a type like InputOutputStream will ever be defined. Even if two types will *never* have a common subtype,<sup>5</sup> the programmer must specify a disambiguating method, one that perhaps throws an exception. Thus, the problem with this approach is that the programmer is required to write numerous additional methods—exponential in the number of incomparable types—some of which may never be called. JPred alleviates the problem somewhat by providing syntax to specify that a particular branch should be preferred in the case of an ambiguity, but it may not always be possible for programmers to know in advance which method to mark as preferred.

Note that neither JPred interfaces nor Fortress traits may contain state and thus the languages do not provide a solution to the object initialization problem; neither does Dubious, since it does not contain constructors.

These solutions and the previously described related work are summarized in Table 1.

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<sup>5</sup>In Fortress, the programmer may specify that two traits are disjoint, meaning that there will never be a subtype of both. To allow modular typechecking, this disjoint specification must appear on one of the two trait definitions, which means that one must have knowledge of the other; consequently this is not an extensible solution.

Language	Object initialization	Multimethod ambiguity
Eiffel	repeat initialization	–
C++	special constructor semantics	–
Scala	no-arg constructors	–
Fortress traits	n/a	disambiguating methods
Stateful traits	repeat initialization	–
Mixins	linearization	–
JPred	n/a	disambiguating methods
Dubious	n/a	MI restrictions

Table 1: Summary of related work and solutions to the object initialization and modular multimethod problems.

### 4.3 Multimethods in CZ

To solve the problem of modular multiple dispatch, we use the same solution as for the object initialization problem: inheritance diamonds are forbidden, and `requires` is used as a substitute. An additional constraint is that a multimethod may only specialize a method in a superclass, not a required class (i.e., specialization is based on subclassing, not subtyping). So, in the CZ hierarchy of Fig. 2, the typechecker will signal an error, since the definitions `seek(Stream@InputStream, long)` and `seek(Stream@OutputStream, long)` are not valid specializations of `seek(Stream, long)`.

Let us suppose for a moment that all classes in Fig. 2 have been defined, except `InputStream`. Accordingly, we would re-write the `seek` methods as follows:

```
class StreamHelper {
  // helper methods
  void seekInput(InputStream s, long pos) { ... }
  void seekOutput(OutputStream s, long pos) { ... }

  // multimethods
  void seek(Stream s, long pos) {}
  void seek(Stream@ConcreteInputStream is, long pos) {
    seekInput(is, pos);
  }
  void seek(Stream@ConcreteOutputStream os, long pos) {
    seekOutput(os, pos);
  }
}
```

(Though these definitions are slightly more verbose than before, syntactic sugar could be provided, particularly for mapping multimethods to helper methods.)

Note that the typechecker does *not* require that a method be provided for “`InputStream && OutputStream`,” unlike JPred and Fortress. If a programmer now defines `InputStream`, but does not provide a new specialization for `seek`, the default implemen-

tation of `seek(Stream)` will be inherited. An specialization for `InputStream` can then be implemented, perhaps one that calls `seekOutput()`. Note that this override need not be defined in `StreamHelper` directly; the method may be defined in one of its subclasses.

Here, it is of key importance that subclassing diamonds are disallowed; because they cannot occur, multimethods can be easily checked for ambiguities. *Subtyping* diamonds do not cause problems, as multimethod specialization is based on *subclassing*.

**Dispatch semantics.** There are two dispatch semantics that can be used for multimethods: asymmetric or symmetric. In asymmetric dispatching, the order of arguments affects dispatch. In particular, earlier arguments are treated as more important when selecting between equally specific methods. This semantics is used in a number of languages, such as Common Lisp and parasitic methods, among others [47, 40, 1, 11, 6].

Other languages employ the symmetric dispatch semantics, where all arguments have equal priority in determining method lookup [15, 44, 17, 35]. Some argue that symmetric dispatch is more intuitive and less error-prone than asymmetric dispatch [17, 35], though this form of dispatch adversely affects information hiding. In particular, a class may not hide the existence of a particular method specialization; this information is needed to correctly perform ambiguity checking of subclasses [35]. For this reason, and to simplify the type system and method lookup rules, CZ multimethod dispatch is asymmetric. However, CZ is compatible with symmetric dispatch; a symmetric-dispatch version of CZ would simply require additional (modular) checks on multimethod definitions. Incidentally, method lookup need not change, as these new ambiguity checks would ensure the same result, regardless of whether asymmetric lookup or symmetric lookup were used. Section 8 describes these issues in more detail.

**Fragments of CZ.** Note that it would be possible to omit multimethods from the language and use the CZ design (as is) for only the object initialization problem. That is, our solution can be used to solve either the object initialization problem, the modular multimethod problem, or both.

## 5 Example: Abstract Syntax Trees

Consider a simple class hierarchy for manipulating abstract syntax trees (ASTs), such as the one in Fig. 3. The original hierarchy is the one on the left, which consists of `ASTNode`, `Num`, `Var`, and `Plus`. An `ASTNode` contains a reference pointing to its parent node, as indicated in the figure. Each of the concrete subclasses of `ASTNode` implements its own version of the abstract `ASTNode.eval()` method.

Suppose we wish to add debugging support to our AST, after the original hierarchy is defined. Each node now additionally has a source location field, `DebugNode.location`. Debugging support, on the right side of the figure, is essentially a new dimension of AST nodes that has a dependency on `ASTNode`. We express this using `requires`. Now, classes like `DebugPlus` can multiply inherit from `ASTNode` and `DebugNode` without creating a subclassing diamond. In particular, `DebugPlus` does *not* inherit two copies of the parent field, because `DebugNode` is a

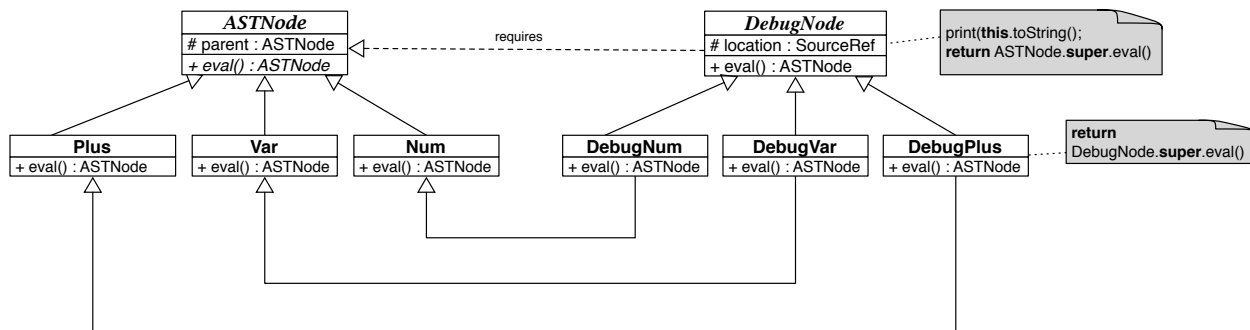


Figure 3: The AST node example in CZ. Abstract classes and abstract methods are set in italic.

```

class DebugNode requires ASTNode {
    ASTNode eval() {
        print(this.toString());
        return ASTNode.super.eval(); // dynamic super call
    }
}

class DebugPlus extends DebugNode, Plus {
    ASTNode eval() {
        return DebugNode.super.eval(); // ordinary super call
    }
}

```

Figure 4: Implementing a mixin-like debug class using dynamically-dispatched super calls, and performing multimethod dispatch on the ASTNode hierarchy.

subtype, but not a subclass, of ASTNode. Thus, the no-diamond property allows fields and multiple inheritance to co-exist gracefully.

In this example, each of these classes has a method `eval()` which evaluates that node of the AST, as in the code in Fig. 4. Suppose we intend `DebugNode` to act as a generic wrapper class for each of the subclasses of `ASTNode`. This can be implemented by using a dynamically-dispatched super call of the form `ASTNode.super.eval()` after performing the debug-specific functionality (in this case, printing the node’s string representation). The prefix `ASTNode.super` means “find the parent class of the dynamic class of this along the `ASTNode` path.” At runtime, when `eval()` is called on an instance of `DebugPlus`, the chain of calls proceeds as follows: `DebugPlus.eval()` → `DebugNode.eval()` → `Plus.eval()`. If the dynamically-dispatched super call behaved as an ordinary super call, it would fail, because `DebugNode` has no superclass.

Each of the `DebugNode` subclasses implements its own `eval()` method that calls `DebugNode.eval()` with an ordinary super call. (This could be omitted if the language linearized method overriding based on the order of inheritance declarations, such as in Scala traits.) Dynamic super calls are a generalization of ordinary super calls, when the qualifier class is a re-



quired class.

**Adding multimethods.** Suppose that after we have defined these classes, we wish to add a new method that operates over the AST. For instance, we may want to check that variables are declared before they are used (assuming a variable declaration statement). Since CZ has multimethods, such a method `defCheck()` could be defined in a helper class, rather than in the classes of the original hierarchy:

```
class DefChecker {
    void defCheck(ASTNode n) { ... }
    void defCheck(ASTNode@Var v) { ... }
    void defCheck(ASTNode@Plus p) { ... }
    void defCheck(ASTNode@Num n) { ... }
}
```

Note that the programmer would *only* have to define cases for `ASTNode`, `Num`, `Var` and `Plus`; she need not specify what method should be called when an object has a combination of these types—such a situation cannot occur (as there are no diamonds).

**Discussion.** The examples illustrate that subtyping allows substitutability; subclassing, in addition to providing inheritance, defines semantic alternatives that may not overlap (such as `Num`, `Var` and `Plus` in the example above). Because they do not overlap, we can safely perform an unambiguous “case” analysis on them—that is, multimethod dispatch. In other words, dispatch in our system is analogous to case-analyzing datatypes in functional programming (e.g. ML, Haskell).

**Alternative designs.** Traits could be used to express this example, but as previously mentioned (Sect. 3.2), the lack of state results in an information hiding problem with accessors. Also, as we have noted, stateful traits do not address the object initialization problem.

Mixins could express some aspects of the class hierarchy for this example, but as previously mentioned, subclassing—or even subtyping—cannot be specified among (standard-style) mixins [12, 24, 4]. For this reason, mixins do not integrate well with multimethods. For details on this issue, see [30].

Using Java-style single inheritance would be unwieldy. A forwarding pattern would have to be used, along with the definition of at least four new interfaces [30]. Additionally, accessors would have to be public, since they would have to be defined in an interface (the only way to achieve any form of multiple inheritance in Java-like languages). Finally, the Visitor design pattern would have to be used in order to allow new operations to be defined.

## 6 CZ Design

In this section, we give informal details of the typechecking rules in CZ, and provide an intuition as to why typechecking is modular. In Sect. 8 we formalize CZ and provide a detailed argument

showing its modularity.

## 6.1 Multiple Inheritance

CZ places the following constraints on class definitions:

- C1.** If a class  $C$  extends  $D_1$  and  $D_2$  then there *must not* exist some  $E$ , other than `Object`, such that both  $D_1$  and  $D_2$  are subclasses of  $E$  (the no-diamond property).
- C2.** If class  $C$  extends  $D_1$  and  $D_2$  and the unspecialized method  $m$  is defined or inherited by both  $D_1$  and  $D_2$  then  $C$  must also define the unspecialized method  $m$ . Also, if method  $m$  with specializer  $B$  is defined or inherited by both  $D_1$  and  $D_2$ , then  $C$  must also define either (1)  $m$  with specializer  $B'$ , where  $B$  is a subclass of  $B'$ , or (2) an unspecialized method  $m$ .

Additionally, the calculus assumes an elaboration phase that translates method names to qualified names, using the name of the class where the method was first defined; consequently, methods have a unique point of introduction. That is, in the calculus, two classes only share a method name if it exists in a common superclass or common required class. This convention prevents a name clash if two methods in unrelated classes  $A$  and  $B$  coincidentally have the same name and a third class inherits from both  $A$  and  $B$ .<sup>6</sup> (Of course, an implementation of the language would have to provide a syntactic way for disambiguating methods that accidentally have the same name; this could be achieved through rename directives, e.g., Eiffel [31], or by using qualified names, e.g., C# interfaces and C++.)

We have already described the reason for condition *C1*, the no-diamond property. We make a special case for the class `Object`—the root of the inheritance hierarchy, since every class automatically extends it. (Otherwise, a class could never extend two unrelated classes—the existence of `Object` would create a diamond.<sup>7</sup>) Note that this does not result in the object initialization problem, because `Object` has only a no-argument constructor. Also, this condition does not preclude a class from inheriting from two concrete classes if this does not form a diamond.

Condition *C2* ensures that if a class  $C$  inherits two identical method definitions, either specialized or unspecialized, this will not lead to an ambiguity; in such a case,  $C$  must provide an overriding definition.

## 6.2 Multiple Dispatch

CZ allows methods to be specialized on a subclass of the first argument's class type. Any unspecialized method (i.e., an ordinary method) defined or inherited by a class  $C$  may be specialized within  $C$ , provided the method's first argument type is not `Object`. In general, typechecking multimethods has two components: *exhaustiveness checking* (i.e., the provided cases provide

---

<sup>6</sup>Incidentally, this is not the convention used in Java interfaces, but is that of C#.

<sup>7</sup>An alternative design would be to make every abstract class implicitly require `Object` and every concrete class implicitly extend `Object`. The problem with this design is that it would prevent a class from extending two concrete classes, as a diamond with `Object` at the root would result.

full coverage of the dispatch hierarchy) and *ambiguity checking* (i.e., when executing a given method call, there is a unique most specific applicable method).

Since the core calculus of CZ does not include abstract methods, exhaustiveness is automatically handled; we need only ensure there are no ambiguities. (Abstract methods are orthogonal to our considerations, as they are adequately handled by previous work [36, 16, 34].) We adapt previous techniques for ambiguity checking [36, 16, 34]:

**MI.** A method  $D\ m(A@B\ x, \overline{D}\ \overline{x})$  may only be defined if in class  $C$  if all of the following hold:

1.  $A \neq \text{Object}$
2.  $B \neq A$
3.  $B$  is a subclass of  $A$
4. a method  $D\ m(A, \overline{D})$  is defined or inherited by  $C$ .

These conditions, together with with  $C1$  and  $C2$ , ensure the absence of ambiguity. In particular, since  $B$  must be a strict subclass of  $A$ , condition  $C1$  ensures that if method  $m(A@B')$  is defined or inherited by  $C$ , then either  $B \leq B'$  or  $B' \leq B$  (since  $A \neq \text{Object}$  and inheritance diamonds are disallowed). Condition  $C2$  ensures that if  $B = B'$ , there exists a disambiguating definition  $m(A@B'')$  within class  $C$ , where  $B \leq B''$ . Together, these properties ensure that if a program typechecks, a unique most-specific applicable method always exists.

Previous work either disallowed inheritance across module boundaries [36] or did not permit interfaces to be specializers [16]. In CZ, we can remove each of these restrictions, due to the absence of inheritance diamonds.

In Sect. 8, we describe a generalization of multimethods to the  $n$ -argument case and describe why this generalization does not introduce new typechecking issues.

### 6.3 Dynamically-Dispatched Super Calls

As illustrated in Sect. 5, CZ includes dynamically-dispatched super calls. When  $A$  requires  $B$  (i.e.,  $A$  is acting as a mixin extension of  $B$ ), then within  $A$ , a call of the form  $B.\text{super}$  is dynamically resolved, similar to super calls in traits. Other super calls (i.e., those where the qualifier is a parent class) have the same semantics as that of Java.

### 6.4 Discussion

**Extensions.** External methods (also known as open classes), could also be added to CZ, without sacrificing modular typechecking. External methods are more general than multimethods, since they allow new classes to override an existing external method. For details on the typechecking issues that arise, see our previous work [30].

It would also be possible to combine our solution with existing techniques for dealing with the object initialization and modular multiple dispatch problems. A programmer could specify that a class  $C$ , whose constructor takes no arguments, may be the root of a diamond hierarchy. Then, we would use the Scala solution for ensuring that  $C$ 's constructor is called only once.

To solve the multiple dispatch problem, if methods  $m(B)$  and  $m(B')$  specialize  $m(C)$ , the type-checker would ensure that  $m$  contained a disambiguating definition for  $(B \wedge B')$ —the JPred and Fortress solutions.

Finally, the language could include syntactic sugar to ease the definition of concrete classes. If  $C$  requires  $B$ , and both  $C$  and  $B$  have no-argument constructors, the compiler could automatically generate a class  $C\$concrete$  that extends both  $C$  and  $B$ ; programmers could then more easily define multimethods that dispatch on  $C\$concrete$ .

**Encapsulation and the diamond problem.** As noted by Snyder, there are two possible ways to view inheritance: as an internal design decision chosen for convenience, or as a public declaration that a subclass is specializing its superclass, thereby adhering to its semantics [46].

Though Snyder believes that it can be useful to use inheritance without it being part of the external interface of a class, we argue that the second definition of inheritance is more appropriate. In fact, if inheritance is being used merely out of convenience (e.g., `Stack` extending `Vector` in the Java standard library), then it is very likely that *composition* is a more appropriate design [10]. For similar reasons, we do not believe a language should allow inheritance without subtyping—e.g., C++ private inheritance—as this can always be implemented using a helper class whose visibility is restricted using the language’s module system.

Nevertheless, if one takes the view that inheritance choices should *not* be visible to subclasses, a form of the diamond problem can arise in CZ. In particular, suppose class  $D$  extends  $B$  and  $C$ ,  $C$  extends  $A$ , and  $B$  extends `Object`—a valid hierarchy (recall that condition  $C1$  makes a special exception for diamonds involving `Object`). Now suppose that  $B$  is changed to extend  $A$ , and the maintainer of  $B$  is unaware that class  $D$  exists. Now  $A$ ,  $B$  and  $C$  typecheck, but  $D$  does not. Thus, the use of inheritance can invalidate subclasses, which violates Snyder’s view of encapsulation.

This situation highlights the fact that, in general, `requires` should be favored over `extends` if a class is intended to be reused.

## 7 Real-World Examples

In this section, we present real-world examples (in both C++ and Java) that suggest that multiple inheritance, and diamond inheritance in particular, can be useful for code reuse. We also describe how these examples can be expressed in CZ.

### 7.1 C++ Examples

We examined several open-source C++ applications in a variety of domains and found many instances of virtual inheritance and inheritance diamonds. Here we describe inheritance diamonds in two applications: Audacity<sup>8</sup> and Guikachu.<sup>9</sup>

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<sup>8</sup><http://audacity.sourceforge.net/>

<sup>9</sup><http://cactus.rulez.org/projects/guikachu/>

**Audacity.** Audacity is a cross-platform application for recording and editing sounds. One of its main storage abstractions is the class `BlockedSequence` (not shown), which represents an array of audio samples, supporting operations such as cut and paste. A `BlockedSequence` is composed of smaller chunks; these are objects of type `SeqBlock`, depicted in Fig. 5 (a). One subclass of `SeqBlock` is `SeqDataFileBlock`, which stores the block data on disk. One superclass of `SeqDataFileBlock` is `ManagedFile`, an abstraction for temporary files that are de-allocated based on a reference-counting scheme. Since both `ManagedFile` and `SeqBlock` inherit from `Storable` (to support serialization), this forms a diamond with `Storable` at the top.

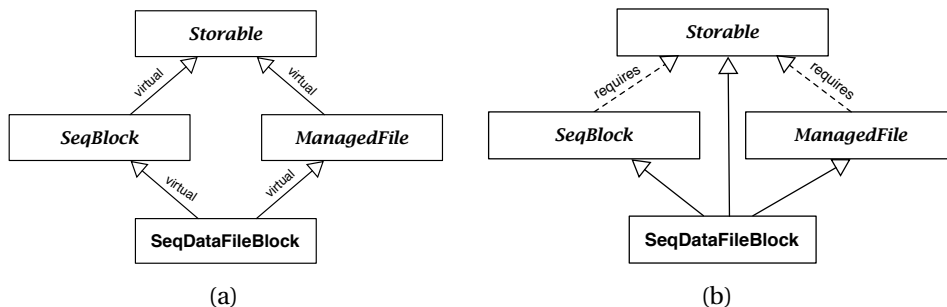


Figure 5: An inheritance diamond (a) in the Audacity application, and (b) the re-written class hierarchy in CZ. Abstract classes are set in italic.

This particular diamond can be easily re-written in CZ (Fig. 5 (b)), since the sides of the diamond (`SeqBlock` and `ManagedFile`) are already abstract classes. (Compare to the example in Fig. 2, where new concrete classes had to be defined for the sides of the diamond.) Here, we simply change the top two virtual inheritance edges to `requires` edges, and make `SeqDataFileBlock` inherit from `Storable` directly. This may even be a preferable abstraction; while in the original hierarchy `SeqDataFileBlock` is serializable by virtue of the fact that `SeqBlock` is serializable, in the new hierarchy we are making this relationship explicit.

**Guikachu.** Guikachu is a graphical resource editor for the GNU PalmOS SDK. It allows programmers to graphically manipulate GUI elements for a Palm application in the GNOME desktop environment. In this application, we found 10 examples of diamonds that included the classes `CanvasItem`, `WidgetCanvasItem`, and `ResizableCanvasItem`. `CanvasItem` is an abstract base class that represents items that can be placed onto a canvas, while objects of type `WidgetCanvasItem` and `ResizableCanvasItem` are a type of widget or are resizable, respectively.

Figure 6(a) shows two of these 10 diamonds, formed by `TextFieldCanvasItem` and `PopupTriggerCanvasItem`, respectively. The hierarchy was likely designed this way because there exist GUI elements that have only one of the two properties. For instance, `GraffitiCanvasItem` and `LabelCanvasItem` (not shown) are not resizable, but they are widgets. In contrast, the class `FormCanvasItem` (not shown) is resizable, but is not a widget.

In this application, we also observed the use of the C++ virtual inheritance initializer invocation mechanism: `TextFieldCanvasItem` (for instance) directly calls the initializer of `CanvasItem`, its grandparent. As previously described, when initializing `TextFieldCanvasItem`, the initializer

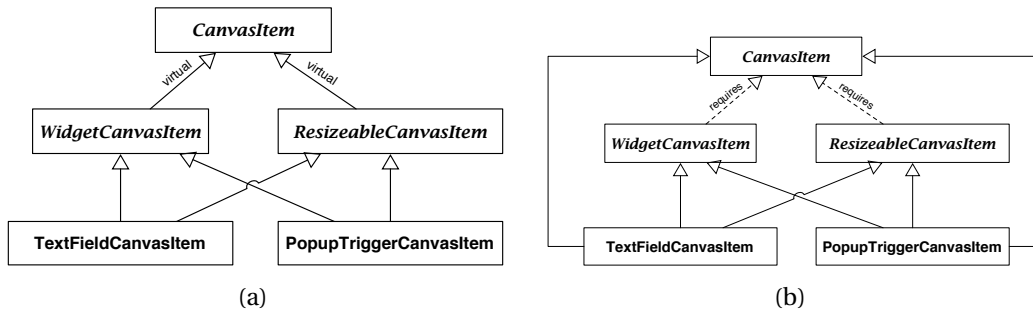


Figure 6: Two inheritance diamonds in the Guikachu application (a) and re-written in CZ (b). Abstract classes are set in italic.

calls from `WidgetCanvasItem` and `ResizeableCanvasItem` to `CanvasItem` are ignored. In this application, the initializers happen to all perform the same operation, but this invocation semantics could introduce subtle bugs as the application evolves.

The corresponding CZ class hierarchy is displayed in Fig. 6 (b); note its similarity to that of Fig. 5 (b). Essentially, the virtual inheritance is replaced with `requires` and each of the classes at the bottom of the diamond inherit from all three of `WidgetCanvasItem`, `ResizeableCanvasItem`, and `CanvasItem`. The CZ design has the advantage that constructor calls do not occur more than one level up the hierarchy, and no constructor calls are ignored.

This example illustrates how a program could be translated from C++-style multiple inheritance to CZ-style. In particular, virtual inheritance would be replaced by `requires`, and new concrete classes would be defined as necessary (changing instantiations of the now-abstract class to instantiations of the new concrete class). Note that constructor calls can be easily generated for the new concrete classes, as C++ requires a call from the bottom of the diamond to the top of the diamond when virtual inheritance is used (such a constructor call would be necessary for the new concrete class, as it would directly extend the class at the top of the diamond).

**Discussion.** It would be interesting to extend the C++ study and perform a more systematic study of the nature of inheritance diamonds, quantifying how often new abstract classes would have to be defined (i.e., how often concrete classes appear on the “sides” of the diamond). One could also determine how often the initializer problem occurs in real code.

However, note that the multimethod problem will always arise in a multiple inheritance situation, even if a programmer never actually creates an inheritance diamond, and (as noted in Section 4) even if a language includes the more benign feature of multiple interface inheritance (e.g., Java-like languages).

## 7.2 Java Example: Eclipse JDT

The Eclipse JDT (Java Development Tools) is an example of where multiple inheritance could be useful for Java programs. In the JDT, every AST node contains *structural properties*. A node’s structural properties allow uniform access to its components. For example, `DoStatement` has 2

fields of type `StructuralPropertyDescriptor`: `EXPRESSION_PROPERTY` and `BODY_PROPERTY`. To get the expression property of a `DoStatement` object, the programmer may call `ds.getExpression()` or `ds.getStructuralProperty( DoStatement.EXPRESSION_PROPERTY)`. Structural property descriptors are often used to specify how AST nodes change when a refactoring is performed.

Through inspection of the JDT code, we found that there was a great deal of duplication among the code for getting or setting a node property using the structural property descriptors. For example, 19 AST classes (e.g., `AssertStatement` and `ForStatement`) have `getExpression/setExpression` properties. As a result, in the method `internalGetSetChildProperty` (an abstract method of `ASTNode`), there are 19 duplications of the following code:

```
if (property == EXPRESSION_PROPERTY) {
    if (get) {
        return getExpression();
    } else {
        setExpression((Expression) child);
        return null;
    }
} else if (property == BODY_PROPERTY) {
    ... // code for body property
}
}
```

Additionally, there are duplicate, identical definitions of the `EXPRESSION_PROPERTY` field. Without a form of multiple inheritance, however, it is difficult to refactor this code into a common location—`DoStatement`, for example, already has the superclass `Statement`. With multiple inheritance, the programmer could create an abstract helper class `ExprPropertyHelper` that requires `ASTNode`. This new class would contain the field definition and an override of `internalGetSetChildProperty`. `DoStatement` would then inherit from both `Statement` and `ExprPropertyHelper` and would have the following body for `internalGetSetChildProperty`:

```
if (property == BODY_PROPERTY) {
    ... // code for body property
} else {
    return ExprPropertyHelper.super.
        internalGetSetChildProperty(property, get, child);
}
```

Additionally, this is a scenario where multiple dispatch would be beneficial. The framework defines various visitors for traversing an AST; these could be omitted in favor of multimethods, which are more extensible.

Overall, our real-world examples suggest that multiple inheritance can be useful, and that even diamond inheritance is used in practice. We have shown that the inheritance diamonds can be easily translated to CZ and that the resulting designs offer some benefits over the original ones. In particular, CZ avoids the problem of ignored constructor calls in C++, while providing more flexible code reuse than with single inheritance.

Declarations	$L ::= \text{class } C \text{ extends } \overline{C} \text{ requires } \overline{C} \{ \overline{C} \overline{f}; K \overline{M} \}$
Constructors	$K ::= C(\overline{C} \overline{f}) \{ \text{this}.\overline{f} = \overline{f}; \}$
Methods	$M ::= C_0 m(\overline{C} \overline{x}) \{ \text{return } e; \} \mid C_0 m(C@C' x, \overline{C} \overline{x}) \{ \text{return } e; \}$
Expressions	$e ::= x \mid e.f \mid e.m(\overline{e}) \mid e.C.\text{super}.m(\overline{e}) \mid \text{new } C(\overline{e})$

Figure 7: CZ grammar

<b>Subclassing</b>	$C \preceq D$
	$\frac{}{C \preceq C}$ $\frac{C \preceq D \quad D \preceq E}{C \preceq E}$ $\frac{CT(C) = \text{class } C \text{ extends } D_1, \dots, D_n \cdots \{ \dots \}}{C \preceq D_i}$
<b>Subtyping</b>	$C <: D$
	$\frac{C \preceq D}{C <: D}$ $\frac{C <: D \quad D <: E}{C <: E}$ $\frac{CT(C) = \text{class } C \text{ extends } \overline{D} \text{ requires } E_1, \dots, E_n \{ \dots \}}{C <: E_i}$

Figure 8: Subclassing ( $\preceq$ ) and subtyping ( $<:$ ) judgement

## 8 Formal System

In this section, we describe the formalization of CZ, which is based on Featherweight Java (FJ) [28]. We use the same conventions as FJ;  $\overline{D}$  is shorthand for the (possibly empty) list  $D_1, \dots, D_n$ , which may be indexed by  $D_i$ .

The grammar of CZ is presented in Fig. 7. Modifications to FJ are highlighted. Class declarations may extend or require a *list* of classes. There is also a new syntactic form for multimethods; such methods include a specializer on the first argument type.

We relax the FJ convention that a class may not define two methods with the same name; such a case is permitted as long as one method or both methods have specializers (which must be distinct). The type of all other arguments and the return type must remain the same.

To simplify the formal system, we assume that all methods have at least one argument. A dummy object can be used to simulate a no-argument method.

To avoid syntax for resolving different superclass constructors, all fields, including those inherited from superclasses, must be initialized in the constructor.

Aside from dynamically-dispatched super calls, and the removal of casts (they are orthogonal to our goals), CZ expression forms are identical to those of FJ. For simplicity, we have not modeled ordinary super calls in our calculus, as this has been considered by others (e.g., [24, 37]) and is orthogonal to the issues we are considering. Therefore, the class qualifier of a super call must be a required class.

We have added a new subclass ( $\preceq$ ) judgement (Fig. 8), which is the reflexive, transitive



$\Gamma \vdash e : C$	
$\frac{}{\Gamma \vdash x : \Gamma(x)}$ <p>(T-VAR)</p>	$\frac{\Gamma \vdash e_0 : C_0 \quad C_0 <: D \quad \text{fields}(D) = \overline{C} \overline{f}}{\Gamma \vdash e_0.f_i : C_i}$ <p>(T-FIELD)</p>
$\frac{\Gamma \vdash e_0 : C_0 \quad \text{mtype}(m, C_0) = \overline{D} \rightarrow C \quad \Gamma \vdash \overline{e} : \overline{C} \quad \overline{C} <: \overline{D}}{\Gamma \vdash e_0.m(\overline{e}) : C}$ <p>(T-INVK)</p>	$\frac{\Gamma \vdash e_0 : C_0 \quad \text{class } C_0 \text{ extends } \overline{D}_0 \text{ requires } B, \overline{E} \quad \text{mtype}(m, B) = \overline{D} \rightarrow C \quad \Gamma \vdash \overline{e} : \overline{C} \quad \overline{C} <: \overline{D}}{\Gamma \vdash e_0.B.\text{super}.m(\overline{e}) : C}$ <p>(T-SUPER-INVK)</p>
$\frac{\text{fields}(C) = \overline{D} \overline{f} \quad \Gamma \vdash \overline{e} : \overline{C} \quad \overline{C} <: \overline{D} \quad \text{class } C \text{ requires } \bullet}{\Gamma \vdash \text{new } C(\overline{e}) : C}$ <p>(T-NEW)</p>	

Figure 9: Expression typing

closure of extends. The subtype judgement ( $<:$ ) is extended to include the requires relationship. Subclassing implies subtyping, and if class  $A$  requires  $B$  then  $A <: B$ , but  $A \not\leq B$ . In CZ, the requires relation is not transitive; subclasses must either require or extend the required class, which is enforced by the typechecking rules. Subtyping allows  $A$  be used in place of  $B$ , which is in contrast to Scala; Scala only allows such a substitution for the `this` reference within a class.

The auxiliary judgements for typechecking appear after the typechecking and evaluation rules, in Fig. 12. We will describe each of these when describing the rules that use them.

**Static Semantics.** The rules for typechecking expressions are in Fig. 9. The rule for method invocations, T-INVK, is the same as that in FJ. However, the auxiliary judgement it uses,  $\text{mtype}$ , is different.

The CZ judgement  $\text{mtype}$  (Fig. 12) has an additional rule as compared to FJ; it performs a lookup of methods from required classes, in the case that the method does not exist in the class itself or superclasses. This judgement considers only unspecialized methods.

The rule T-SUPER-INVK checks the dynamically-dispatched super call described in Sect. 6. Essentially, for a call of the form `this.B.super.m( $\overline{e}$ )`, where `this` :  $C_0$ , instead of looking up  $\text{mtype}(m, C_0)$ , we look up  $\text{mtype}(m, B)$ , where  $B$  is a required class of  $C_0$ .

The rule T-NEW has one additional premise as compared to FJ: the requires clause must be empty. This ensures that the class is concrete and can be instantiated, which in turn ensures the soundness of the subtyping relation induced by requires.

Rules for typechecking methods are displayed in Fig. 10. The rule T-METHOD checks unspecialized methods, and uses the *override* auxiliary judgement (which is unchanged from FJ). In this rule, we check that method  $m$  is a valid override of the same (unspecialized) method in all superclasses and required classes.

T-MULTI-METHOD checks specialized methods. The first two premises are the same as that of T-METHOD. Premises (3), (4) and (5) check conditions 1, 2, and 3, respectively, of constraint  $MI$ .

<b>M ok in C</b>
$ \begin{array}{c} \textcircled{1} \bar{x} : \bar{B}, \text{this} : C \vdash e_0 : E_0 \quad \textcircled{2} E_0 <: B \quad \textcircled{3} \text{class } C \text{ extends } \bar{D} \text{ requires } \bar{E} \\ \textcircled{4} \forall i. \text{override}(m, D_i, \bar{B} \rightarrow B) \quad \textcircled{5} \forall i. \text{override}(m, E_i, \bar{B} \rightarrow B) \\ \hline B \ m(\bar{B} \ \bar{x}) \{ \text{return } e_0; \} \ \mathbf{ok} \ \text{in } C \quad (\text{T-METHOD}) \end{array} $
$ \begin{array}{c} \textcircled{1} x : B', \bar{x} : \bar{B}, \text{this} : C \vdash e_0 : E_0 \quad \textcircled{2} E_0 <: B \\ \textcircled{3} A \neq \text{Object} \quad \textcircled{4} B_0 \neq A \quad \textcircled{5} B_0 \leq A \quad \textcircled{6} \text{mtype}(m, C) = (A, \bar{B}) \rightarrow B \\ \hline B \ m(A@B_0 \ x, \bar{B} \ \bar{x}) \{ \text{return } e_0; \} \ \mathbf{ok} \ \text{in } C \quad (\text{T-MULTI-METHOD}) \end{array} $

Figure 10: Specialized and unspecialized method typing

<b>Declaration Typing</b> <b>L ok</b>
$ \begin{array}{c} \textcircled{1} \forall i. \text{fields}(D_i) = \bar{F}_i \ \bar{g}_i \quad \textcircled{2} K = C(\bar{F}_i \ \bar{g}_i^{i \in 1..n}, \bar{C} \ \bar{f}) \{ \text{this}.\bar{g}_i = \bar{g}_i^{i \in 1..n}; \text{this}.\bar{f} = \bar{f} \} \\ \textcircled{3} \bar{M} \ \mathbf{ok} \ \text{in } C \quad \textcircled{4} \forall i. \text{class } D_i \text{ requires } E', \text{ implies } \exists k. D_k \leq E' \text{ or } \exists k. E_k \leq E' \\ \textcircled{5} \forall i. \text{class } E_i \text{ requires } E'', \text{ implies } \exists k. D_k \leq E'' \text{ or } \exists k. E_k \leq E'' \\ \textcircled{6} \forall i. \forall j \neq i. \bar{A} D' \neq \text{Object}. D_i \leq D' \text{ and } D_j \leq D' \\ \textcircled{7} \forall m. \forall B'. (\exists i. \exists j \neq i. \text{methodDef}(m, D_i, B') \text{ and } \text{methodDef}(m, D_j, B')), \text{ implies} \\ \exists B''. B' \leq B'' \text{ and } B_0 \ m(\langle A@ \rangle B'' \ x, \bar{B} \ \bar{x}) \in \bar{M} \\ \hline \text{class } C \text{ extends } \bar{D} \text{ requires } \bar{E} \ \{ \bar{C} \ \bar{f}; K \ \bar{M} \} \ \mathbf{ok} \quad (\text{T-CLASS}) \end{array} $

Figure 11: Class typing

Premise (6) checks condition 4 of *MI*; it ensures *C* defines or inherits an unspecialized method with type  $(A, \bar{B}) \rightarrow B$ , where *A* is the static type being specialized.

The T-CLASS rule (Fig. 11) checks class definitions. Premises (1–3) are straightforward generalizations of the corresponding premises in FJ. Premises (4) and (5) ensure that *requires* is propagated down each level of the inheritance hierarchy; the extending class must either extend or require its parents' required classes. Premise (6) specifies that a subclassing diamond cannot occur, except for the case of *Object* (condition *C1*). Finally, premise (7) enforces condition *C2*, ensuring that if *C* inherits two methods *m* with the same first argument *B'* (or two unspecialized methods *m*), then *C* provides an overriding definition for *m*. This premise uses the *methodDef*(*m*, *D<sub>i</sub>*, *B'*) auxiliary judgement: a derivation of *methodDef* exists if *D<sub>i</sub>* defines or inherits a method with specializer *B'* or first argument type *B'*. This premise, as well as the *methodDef* judgement, uses the notation  $\langle A@ \rangle$  to specify either a specialized or unspecialized method (i.e., the *A@* part is optional).

**Dynamic Semantics.** The evaluation rules and auxiliary judgements are presented in Fig. 13. Most of the rules are similar to FJ, with the exception of E-INVK and E-SUPER-INVK. E-INVK passes

$$\boxed{\text{fields}(C) = \overline{C} \overline{f}}$$

$$\frac{\text{class } C \text{ extends } D_1, \dots, D_n \text{ requires } \overline{E} \{ \overline{C} \overline{f}; K \overline{M} \} \quad \forall i. \text{fields}(D_i) = \overline{B}_i \overline{g}_i}{\text{fields}(\text{Object}) = \bullet \quad \text{fields}(C) = \overline{B}_i \overline{g}_i^{i \in 1..n}, \overline{C} \overline{f}}$$

$$\boxed{\text{mtype}(m, C) = \overline{D} \rightarrow D}$$

$$\frac{\text{class } C \dots \{ \overline{C} \overline{f}; K \overline{M} \} \quad B_0 m(\langle B' @ \rangle B x, \overline{B} \overline{x}) \{ \text{return } e; \} \in \overline{M}}{\text{mtype}(m, C) = \overline{B} \rightarrow B}$$

$$\frac{\text{class } C \text{ extends } \overline{D} \text{ requires } \overline{E} \{ \overline{C} \overline{f}; K \overline{M} \} \quad \exists k. \text{mtype}(m, D_k) = \overline{B} \rightarrow B}{\text{mtype}(m, C) = \overline{B} \rightarrow B}$$

$$\frac{\text{class } C \text{ extends } \overline{D} \text{ requires } \overline{E} \{ \overline{C} \overline{f}; K \overline{M} \} \quad \exists k. \text{mtype}(m, E_k) = \overline{B} \rightarrow B}{\text{mtype}(m, C) = \overline{B} \rightarrow B}$$

$$\boxed{\text{methodDef}(m, C, B)}$$

$$\frac{\text{class } C \dots \{ \overline{C} \overline{f}; K \overline{M} \} \quad B_0 m(\langle B' @ \rangle B x, \overline{B} \overline{x}) \{ \text{return } e; \} \in \overline{M}}{\text{methodDef}(m, C, B)}$$

$$\frac{\text{class } C \text{ extends } \overline{D} \dots \{ \overline{C} \overline{f}; K \overline{M} \} \quad B_0 m(\langle B' @ \rangle B x, \overline{B} \overline{x}) \{ \text{return } e; \} \notin \overline{M} \quad \exists k. \text{methodDef}(m, D_k, B)}{\text{methodDef}(m, C, B)}$$

$$\boxed{\text{override}(m, D, \overline{C} \rightarrow C_0)}$$

$$\frac{\text{mtype}(m, D) = \overline{D} \rightarrow D_0 \text{ implies } \overline{C} = \overline{D} \text{ and } C_0 = D_0}{\text{override}(m, D, \overline{C} \rightarrow C_0)}$$

Figure 12: CZ typechecking auxiliary judgements

the dynamic type of the method's first argument as an additional argument to *mbody*, which we describe below. E-SUPER-INVK uses the auxiliary judgement  $\text{super}(C, D)$ , which finds the first superclass of the class  $C$  that is also a subclass of  $D$ . Then, *mbody* is called on the result of the *super* call.

The main changes to the dynamic semantics are encoded in the auxiliary judgements *mbody*, *dispatch*, and *match*. The *mbody* judgement has one additional argument as compared with FJ, to (potentially) dispatch on the method's first argument. This judgement simply extracts the arguments and method body from the result of the *dispatch* judgement, which contains the actual dispatch logic.

Method dispatch is performed in two steps. The first *dispatch* rule uses the *matchArg* judgement to search for a method defined in  $C$  that is applicable for  $D$ , the dynamic type of the

method’s first argument. This latter judgement considers both specialized and unspecialized methods (via the  $\langle B@ \rangle$  notation). If such a definition exists, it is returned; otherwise, a set of methods  $\overline{M}_E$  is composed by calling *dispatch* on each of  $C$ ’s superclasses. Then, the unique most specific method that is applicable for argument  $D$  is selected. Note the asymmetric dispatch semantics; if an appropriate method does not exist in  $C$ , its superclasses are searched before dispatching on the argument type.

**Constructors.** As in FJ, CZ does not contain state, and thus constructor definitions are trivial. However, a full implementation would have to ensure that when  $C$  extends  $A, B$  and  $A$  requires  $B$ , when creating a  $C$  object, its  $B$ -part must be initialized before its  $A$ -part. Otherwise, this could result in fields being accessed before they exist, since  $A$  is permitted to access  $B$ ’s fields.

**Generalizations.** To generalize multimethod dispatch to  $n$  arguments, in the static semantics, we would extend premise (7) of T-CLASS, which corresponds to condition C2. In particular, this premise would ensure that if  $C$  inherits two method definitions that have identical specializer *lists* (or argument types), then an overriding definition exists in  $C$ . In particular, the quantifier  $\forall B'$  would become  $\forall \overline{B}'$  in this premise.

For the dynamic semantics, we would change *dispatch* and *matchArg* to take a *list* of dynamic types of the objects passed to the method in question. The latter judgement would then select the unique most specific method such that each of its specializers (or argument types, if there is no specializer) is a supertype of the corresponding dynamic type. This could be implemented by first creating a candidate list of all applicable methods, then selecting the most specific one.

We observe from the form of these judgements (*dispatch* and *matchArg*) that there could be two ways in which more than one method applies: (1) within a single argument position, more than one method applies and none is more specific than the others, (2) one method is more specific at one argument position and another method is more specific at some other argument position. Note that, in the absence of appropriate typechecking, either situation can arise, *regardless* of whether dispatch is performed on  $n$  arguments or just two arguments. Premise (6) of T-MULTI-METHOD ensures that there exists at least one method in the candidate list.

CZ’s static semantics—in particular, conditions C1 and C2—ensure that the first situation cannot arise. As a consequence of condition C1 (the no-diamond restriction), for a particular argument position  $k$ , all specializer types in the candidate method list are mutually comparable (via subclassing). That is, if, at argument position  $k$ , method  $m_i$  has specializer  $C_i$  and  $m_j$  has specializer  $C_j$ , then either  $C_i \leq C_j$  or  $C_j \leq C_i$  (for  $i \neq j$ ). This is because both types must be superclasses of  $D_k$  (the dynamic type at position  $k$ ) and they also must both be subtypes of  $C_k$ , the static type at position  $k$ . Since  $C_k$  cannot be Object,  $C_i$  and  $C_j$  must be comparable types.

C2 ensures that there exists an argument position  $k$  such that  $C_i \neq C_j$ . We observe that two methods in the candidate list could only have identical specializer types (or argument types) if class  $C$  inherited two such methods (as there is a syntactic restriction against such a definition directly in  $C$ ). But, C2 ensures that if such a situation were to occur, that  $C$  would have an

Evaluation $e \mapsto e'$		
<p>(E-FIELD)</p> $\frac{\text{fields}(C) = \bar{B} \bar{f}}{(\text{new } C(\bar{e})).f_i \mapsto e_i}$	<p>(E-INVK)</p> $\frac{\text{mbody}(m, C, D) = (x, \bar{x}).e_0}{(\text{new } C(\bar{e})).m(\text{new } D(\bar{e}'), \bar{d}) \mapsto [\text{new } C(\bar{e})/\text{this}, \text{new } D(\bar{e}')/x, \bar{d}/\bar{x}] e_0}$	<p>(E-SUPER-INVK)</p> $\frac{\text{super}(C, E) = C' \quad \text{mbody}(m, C', D) = (x, \bar{x}).e_0}{(\text{new } C(\bar{e})).E.\text{super}.m(\text{new } D(\bar{e}'), \bar{d}) \mapsto [\text{new } C(\bar{e})/\text{this}, \text{new } D(\bar{e}')/x, \bar{d}/\bar{x}] e_0}$
$\frac{e_0 \mapsto e'_0}{e_0.f \mapsto e'_0.f}$	$\frac{e_0 \mapsto e'_0}{e_0.m(\bar{e}) \mapsto e'_0.m(\bar{e})}$	$\frac{e_0 \mapsto e'_0}{e_0.C.\text{super}.m(\bar{e}) \mapsto e'_0.C.\text{super}.m(\bar{e})}$
$\frac{e_i \mapsto e'_i}{e_0.m(\dots, e_i, \dots) \mapsto e_0.m(\dots, e'_i, \dots)}$	$\frac{e_i \mapsto e'_i}{e_0.C.\text{super}.m(\dots, e_i, \dots) \mapsto e_0.C.\text{super}.m(\dots, e'_i, \dots)}$	$\frac{e_i \mapsto e'_i}{\text{new } C(\dots, e_i, \dots) \mapsto \text{new } C(\dots, e'_i, \dots)}$
Auxilliary Judgements		
$\text{mbody}(m, C, D) = \bar{x}.e$	$\frac{\text{dispatch}(m, C, D) = B_0 m(\langle B@ \rangle B' x, \bar{B} \bar{x}) \{ \text{return } e; \}}{\text{mbody}(m, C, D) = \bar{x}.e}$	
$\text{super}(C, D) = E$	$\frac{\text{class } C \text{ extends } E \quad E \leq D}{\text{super}(C, D) = E}$	
	$\frac{\nexists E' \leq D. \text{class } C \text{ extends } E' \quad C \text{ extends } \bar{B} \quad \exists k. \text{super}(C, B_k) = E}{\text{super}(C, D) = E}$	
$\text{dispatch}(m, C, D) = M$	$\frac{\text{class } C \dots \{ \bar{C} \bar{f}; K \bar{M} \} \quad \text{matchArg}(m, D, \bar{M}) = M'}{\text{dispatch}(m, C, D) = M'}$	
	$\frac{\text{class } C \text{ extends } \bar{E} \text{ requires } \bar{F} \{ \bar{C} \bar{f}; K \bar{M} \} \quad \nexists M'. \text{matchArg}(m, D, \bar{M}) = M' \quad \bar{M}_E = \{ M_i \mid \text{dispatch}(m, E_i, D) = M_i \} \quad \exists \text{ unique } M''. \text{matchArg}(m, D, \bar{M}_E) = M''}{\text{dispatch}(m, C, D) = M''}$	
$\text{matchArg}(m, D, \bar{M}) = M$	$\frac{M' = B_0 m(\langle B@ \rangle D x, \bar{B} \bar{x}) \{ \text{return } e; \} \quad M' \in \bar{M}}{\text{matchArg}(m, D, \bar{M}) = M'}$	
	$\frac{B_0 m(\langle B@ \rangle D x, \bar{B} \bar{x}) \notin \bar{M} \quad \text{class } D \text{ extends } \bar{E} \quad \exists \text{ unique } (k, M'). \text{matchArg}(m, E_k, \bar{M}) = M'}{\text{matchArg}(m, D, \bar{M}) = M'}$	

Figure 13: Evaluation rules and auxiliary judgements

overriding definition. Therefore, the first *dispatch* rule would apply and superclasses would not be considered.

Finally, situation (2) cannot occur, due our asymmetric dispatch semantics. To change to symmetric dispatch (described in Sect. 4.3), we need only add an additional premise to T-CLASS. This new premise would ensure that there is no combination of receiver and argument tuples such that more than one method would apply, using the same modular check implemented in, for instance, MultiJava and EML [17, 34]. Note that the dynamic semantics would not need to change, since this new premise would ensure that asymmetric and symmetric dispatch produce the same result.

## 8.1 Modularity

Here, we describe the conditions under which a class-based system is modular when there is no explicit module system. We argue informally that typechecking in CZ is modular based on the structure of the typechecking rules. (The other languages we have mentioned also perform modular typechecking by this definition.)

### Conditions for modular typechecking.

1. Checking a class signature  $C$  with methods  $\overline{M}$  should only require examining: (a) signatures of methods transitively overridden or specialized in  $\overline{M}$ , (b) signatures of methods transitively overridden or specialized by  $C$ 's inherited methods, (c) class declarations of  $C$ 's supertypes.
2. Checking the definition of a particular method  $m$  (possibly specialized with class  $C$ ) should only require examining: (a) the declarations of  $C$  and its supertypes, (b) the signature of the method that  $m$  specializes, and (c) the signatures of methods called by  $m$ .

By inspection, checking a class definition  $C$  obeys condition 1. Each premise examines only superclasses or required classes, and there is, for example, no search for multimethods with first argument type  $C$ .

Checking a method definition  $m$  is also modular. If  $m$  is an unspecialized method, the only generalization to the typechecking rule is additional *override* checks, which are modular. On the other hand, when a specialized method is checked, we simply ensure that the specializer has the appropriate relationship to its static type (which may not be Object), and call  $mtype(m, C)$ . Since this judgement only searches up the subtype hierarchy, it is modular.

## 8.2 Type Safety

We prove type safety using the standard progress and preservation theorems, with a slightly stronger progress theorem than that of FJ, due to the omission of casts. Note that in our system, type safety implies that all method calls are unambiguous, as the *dispatch* and *match* judgements require that there be a unique most-applicable method. We describe below a brief outline of the proof of type safety and refer the reader to Appendix B for further details.

**Theorem 8.1** (Preservation). If  $\Gamma \vdash e : C$  and  $e \mapsto e'$ , then  $\Gamma \vdash e' : C'$  for some  $C' <: C$ .

The proof of preservation is relatively straightforward and is similar to the proof of FJ. We make use of an auxiliary lemma (not shown) that proves that *mtype* returns a unique value. The proof of this lemma makes use of the convention that method introductions are unique.

**Theorem 8.2** (Progress). If  $\cdot \vdash e : C$  then either  $e$  is a value or there is an  $e'$  with  $e \mapsto e'$ .

The proof of progress is slightly more complex. The proof requires the following lemma:

**Lemma 8.1.** If  $mtype(m, C) = (B_0, \overline{B}) \rightarrow B$  and  $\Gamma \vdash \text{new } C(\overline{e}) : C$  and  $B' \preceq B_0$  then  $dispatch(m, C, B') = M$ , for some  $M$ .

However, unlike in FJ, we cannot prove this lemma by induction on the derivation of *mtype*, since for the inductive step, we do not have a derivation  $\Gamma \vdash \text{new } D_k(\overline{e}) : D_k$ . Instead, we make use of two auxiliary lemmas:

**Lemma 8.2.** If  $\mathcal{D} :: mtype(m, C) = (B_0, \overline{B}) \rightarrow B$  and  $\mathcal{D}$  does not contain the rule MTYP3 and  $B' \preceq B_0$ , then  $dispatch(m, C, B') = M$ , for some  $M$ .

**Lemma 8.3.** If  $\Gamma \vdash \text{new } C(\overline{e}) : C$  and  $C <: D$  and  $\mathcal{D} :: mtype(m, D) = \overline{B} \rightarrow B$ , then there exist  $D'$  and  $\mathcal{D}'$  such that  $C \preceq D'$  and  $\mathcal{D}' :: mtype(m, D') = \overline{B} \rightarrow B$  does not contain the rule MTYP3.

Lemma 8.2 is needed because it is the rule MTYP3 that could result in *mbody* not being defined—it is the only rule that has no *dispatch* counterpart. We use Lemma 8.3 to produce such an *mtype* derivation.

With these lemmas, the rest of the proof of progress is straightforward.

## 9 Related Work

Here we describe related work that was not previously discussed in Sections 2, 3.2, and 4.2.

As mentioned in Sect. 4.2, JPred [25] and Fortress [3] perform modular multimethod type-checking by requiring that programmers provide disambiguating methods, some of which may never be called. However, we observe that the JPred and Fortress dispatch semantics may be more expressive than that of CZ. In CZ, in the class hierarchy Fig. 2, the abstract class `InputStream` may not be used as a specializer for `Stream`, because it is not a subclass of `Stream`. In contrast, if this hierarchy were expressed in e.g. Fortress a multimethod defined on `Stream` could be specialized for either `InputStream` or `OutputStream`. Note, however, that programmers can achieve a similar effect in CZ by having concrete classes call helper methods (which may themselves perform multiple dispatch) defined on the abstract classes.

Cecil [14, 15] also provides both multiple inheritance and multimethod dispatch, but it does not include constructors (and therefore provides ordinary dispatch semantics for methods acting as constructors), and it performs whole-program typechecking.

Like JPred, the language Half & Half [6] provides multimethod dispatch on Java interfaces. In this language, if there exist specialized method implementations for two incomparable interfaces  $A$  and  $B$ , the visibility of one of the two interfaces must be package-private. Like System M, this effectively disallows multiple (interface) inheritance across module boundaries (where

a package is a module). Half & Half does not consider the problem of multiple inheritance with state.

Pirkelbauer et al have considered the problem of integrating multimethods into C++, which is especially difficult due to existing rules for overload resolution [41]. However, this proposal is not modular; because of the potential for inheritance diamonds, the design requires link-time typechecking.

It is worth noting that multimethods *cannot* be simulated with C# 3.0 “extension methods” or partial classes [33]. The former, extension methods, are merely syntactic sugar and cannot be overridden with a more specific type for the receiver. Partial classes, on the other hand, are simple a compile-time mechanism for splitting a class’s definition across multiple compilation units. In particular, compared to multimethods, they have the following limitations: 1) they cannot span assemblies (so if the AST node classes are in a library, some other mechanism would be needed, such as the Visitor pattern); 2) partial classes may not be used to perform dispatch on interfaces, in contrast to multimethods; and 3) typechecking each part of a partial class is not modular, as all parts are composed before typechecking. This last problem can cause compilation errors if two programmers implement a partial class in incompatible ways, so it is unclear what should be the appropriate level of granularity when partial classes are used in a team environment.

## 10 Conclusions

We have presented a language that solves two major problems caused by inheritance diamonds: object initialization and modular typechecking of multiple dispatch. We have also shown how programs written with traditional multiple inheritance can be converted to programs in our language. We note that though diamonds can still cause encapsulation problems (depending on the definition of encapsulation), this problem can be ameliorated by preferring `requires` over `extends`.

We emphasize that although programmers may indeed have to decide ahead of time whether they want to make a class re-usable by making it abstract and by using `requires` instead of `extends`—potentially a difficult decision to make—it is a decision the class designer must already make, as a class must be designed carefully if it is to be a unit of reuse (e.g., see item 17 in [32]).

One might also raise the objection that CZ would result in a proliferation of abstract classes, for which a corresponding concrete class would have to be defined. We believe that this problem can mostly be solved through a syntactic sugar for defining concrete classes (Section 6.4). Additionally, note that our proposed solution requires just as many abstract classes as there would be mixins or traits (which also cannot be instantiated) if those solutions were to be used (Section 3.2).



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## A Subtyping vs. Subclassing

In CZ, the use of `requires` provides subtyping without inheritance, but it also places constraints on concrete subclasses—they must inherit from their parent’s required classes. This raises the question of whether simply providing subtyping without inheritance would be sufficient to encode the desired relationships.

When separating subtyping from inheritance, we may use nominal subtyping or structural subtyping. However, in either case, private members are problematic. If private members are included in a subtyping relationship, this can violate information hiding, if they are not, it can restrict expressiveness.

Concretely, consider the following program:

```
class A {
  private int i;
  boolean equals(A other) {
    ... // can access other.i?
  }
}

class B subtypes A {
  ... // declare i?
}
```

Suppose that the `subtypes` keyword provides nominal subtyping without inheritance (but without the additional constraints of `requires`). The question then arises: are private members considered when checking subtyping? If so, then `B` must declare a private field `i`. Unfortunately, this also means that `A.equals` can access `B.i`, which violates information hiding; one class should not be able to access private members defined in another class. On the other hand, if we assume that subtyping does not include private members, then `A.equals` cannot access `other.i`, which is problematic if the definition of equality depends on this field. An analogous problem occurs if structural subtyping is used.

The problem can be avoided if inheritance or `requires` is used for types that contain binary methods. Since `requires` is tied to a particular class, if we change the above code to `B requires A` (or `B extends A`), then `A.equals(A other)` may safely access `other.i`, even if an object of type `B` is passed to this method. Note that an information hiding problem does not arise here—the private state has not been redefined in `B`, but is rather (eventually) inherited from `A` in the concrete `B` implementation that was passed in.

## B Type Safety Proof

### B.1 Auxiliary Lemmas

**Lemma B.1.** If  $mtype(m, D) = \bar{C} \rightarrow C_0$ , then for  $C \preceq D$ ,  $mtype(m, C) = \bar{C} \rightarrow C_0$ .

*Proof.* By induction on  $C \preceq D$ .

**case** SUB-CREFL. Immediate.

**case** SUB-CTRANS. We have  $C \preceq D$  and  $D \preceq E$ . By the induction hypothesis,  $mtype(m, D) = \bar{C} \rightarrow C_0$ . Applying the induction hypothesis to  $C \preceq D$  gives the required result.

**case** SUB-EXTENDS. There are two cases:

$C$  defines  $m$ . By inversion on *override* and T-METHOD,  $m$  must be a valid override and must have type  $\bar{C} \rightarrow C_0$ . By rule MTYPE1,  $mtype(m, C) = \bar{C} \rightarrow C_0$ .

$C$  does not define  $m$ . By rule MTYPE2,  $mtype(m, C) = mtype(m, D)$ .

□

**Lemma B.2.** If  $C <: E$  and  $mtype(m, E) = \bar{B} \rightarrow B$ , then  $mtype(m, C) = \bar{B} \rightarrow B$ .

*Proof.* By case analysis of  $C <: D$ .

**case** SUB-SUBCLASS. Follows from Lemma B.1.

**case** SUB-TRANS.  $C <: D$  and  $D <: E$ .

By the induction hypothesis on  $D <: E$ ,  $mtype(m, D) = \bar{B} \rightarrow B$ . The result then follows from the induction hypothesis on  $C <: D$ .

**case** SUB-REQUIRES. There are two cases:

$C$  defines  $m$ . Similar to the same case in Lemma B.1

$C$  does not define  $m$ . By rule MTYPE3,  $mtype(m, C) = mtype(m, E)$ .

□

**Lemma B.3.** If  $mtype(m, C) = \bar{B} \rightarrow B$  and  $mtype(m, C') = \bar{B}' \rightarrow B'$ , then there exists a  $D$  where  $C <: D$  and  $C' <: D$  and  $mtype(m, D) = \bar{B}'' \rightarrow B''$  where  $\bar{B} = \bar{B}' = \bar{B}''$  and  $B = B' = B''$ .

*Proof.* By simultaneous induction on the two  $mtype$  derivations.

**case** MTYPE1, MTYPE1. By the convention that methods have a unique point of introduction,  $B'' \ m(\bar{B}'' \ \bar{x})$  must have been introduced in some  $D$  where  $C <: D$  and  $C' <: D$ . By MTYPE1,  $mtype(m, D) = \bar{B}'' \rightarrow B''$ . The result then follows from Lemma B.2.

**case** —, MTYPE2; —, MTYPE3. The result follows from the induction hypothesis and the transitivity of subtyping.

□

**Lemma B.4.** If  $A \leq B$  and  $B$  requires  $C$ , then either there exists some  $C' \leq C$  such that  $A$  requires  $C'$  or  $A \leq C'$ .

*Proof.* Straightforward induction on  $A \leq B$ .

□

**Lemma B.5.** If  $A <: B$  and  $A \not\leq B$  then there exists some  $B' \leq B$  such that  $A$  requires  $B'$ .

*Proof.* By induction on  $A <: B$ .

**case** SUB-SUBCLASS. Vacuous.

**case** SUB-TRANS. We have  $A \leq C$  and  $C \leq B$ .

Since  $A \not\leq B$ , there are three possibilities:

**subcase**  $A \not\leq C, C \not\leq B$ . By the induction hypothesis on the first derivation, we have  $\exists C' \leq C$ .  $A$  requires  $C'$ . By the induction hypothesis on the second derivation,  $\exists B' \leq B$ .  $C$  requires  $B'$ . We have  $C' \leq C$  and  $C$  requires  $B'$ . Taking these facts together, by Lemma B.4,  $\exists B'' \leq B'$ .  $C'$  requires  $B''$  or  $C' \leq B''$ . In the first case, again by Lemma B.4,  $\exists B''' \leq B'$ .  $A$  requires  $B'''$ . But, since  $B''' \leq B$ , this proves the required result.

**subcase**  $A \leq C, C \not\leq B$ . By the induction hypothesis,  $\exists B' \leq B$ .  $C$  requires  $B'$ . Since  $A \leq C$ , by Lemma B.4,  $\exists B'' \leq B'$ .  $A$  requires  $B''$  or  $A \leq B''$ .

In the first case,  $A$  requires  $B''$ , the result follows from the fact that  $B'' \leq B$ . In the second case,  $A \leq B''$ , we have  $A \leq B$ , which is a contradiction.

**subcase**  $A \not\leq C, C \leq B$ , by the induction hypothesis,  $\exists C' \leq C$ .  $A$  requires  $C'$ . The result follows from the fact that  $C' \leq B$ .

**case** SUB-REQUIRES. Immediate. □

**Lemma B.6.** If  $matchArg(m, A, \overline{M}) = M_0$  and  $arg_I(M_0) = A'$  then  $M_0 = B_0 m(\langle B @ \rangle A' x, \overline{B} \overline{x})$  and  $M_0 \in \overline{M}$  and  $A \leq A'$ .

*Proof.* By induction on  $matchArg$ .

**case** MATCH1. Immediate. □

**case** MATCH2. We have  $matchArg(m, A_k, \overline{M}) = M_0$ , where  $A$  extends  $A_k$ . By the induction hypothesis,  $M_0 \in \overline{M}$  and  $A_k \leq A'$ . The result then follows from the transitivity of subclassing. □

**Lemma B.7.** If  $arg_I(M_0) = A'$  and  $M_0 \in \overline{M}$  and  $A \leq A'$ , then  $matchArg(m, A, \overline{M}) = M'_0$  where  $arg_I(M'_0) \leq A'$ .

*Proof.* By induction on  $A \leq A'$ .

**case** SUB-REFL. Immediate from MATCH1. □

**case** SUB-CTRANS. We have  $A \leq B$  and  $B \leq A'$ . By the induction hypothesis on the second derivation,  $matchArg(m, B, \overline{M}) = M'_0$  where  $arg_I(M'_0) \leq A'$ . By Lemma B.6,  $M'_0 \in \overline{M}$ . By the induction hypothesis on the first derivation ( $A \leq B$ ),  $matchArg(m, A, \overline{M}) = M''_0$  and  $arg_I(M''_0) \leq arg_I(M'_0)$ . By transitivity of subtyping,  $arg_I(M''_0) \leq A$ , which is the required result. □

**case** SUB-EXTENDS. We have  $A$  extends  $A'$ . Either (1) there exists  $M'$  where  $arg_I(M') = A'$  and  $M' \in \overline{M}$  or (2)  $M' \notin \overline{M}$ . In case (1), by MATCH1,  $matchArg(m, A, \overline{M}) = A$ . Otherwise, in case (2), the result follows from MATCH2. □

## B.2 Progress Lemmas and Proof

**Definition B.1.**  $arg_I(B \ m(\langle A@ \rangle B_0 \ x, \overline{B} \ \overline{x}) \{ \text{return } e; \}) \stackrel{\text{def}}{=} B_0$

**Lemma B.8** (No-diamond property). If  $A \leq D_1$  and  $A \leq D_2$  and  $D_1 \leq B$  and  $D_2 \leq B$ , then either  $B = \text{Object}$  or  $D_1 \leq D_2$  or  $D_2 \leq D_1$ .

*Proof.* By simultaneous induction on  $A \leq D_i$  and  $A \leq D_j$

**case** SUB-CREFL, SUB-CREFL.  $D_1 = D_2$ . Immediate.

**case** SUB-CREFL, SUB-CTRANS; SUB-CREFL, SUB-EXTENDS.  $A = D_1$ . By assumption,  $A \leq D_2$ , which is the required result.

**case** SUB-CTRANS, SUB-CTRANS. We have  $A \leq C_1$  and  $C_1 \leq D_1$  and  $A \leq C_2$  and  $C_2 \leq D_2$ . By the transitivity of subclassing,  $C_1 \leq B$  and  $C_2 \leq B$ . Applying the induction hypothesis to  $A \leq C_1$  and  $A \leq C_2$ , we have either (1)  $B = \text{Object}$  or (2)  $C_1 \leq C_2$  or (3)  $C_2 \leq C_1$ . In case (1), the result follows. In case (2), we have  $C_1 \leq D_1$  and  $C_1 \leq D_2$ . The result follows by applying the induction hypothesis to these derivations. Case (3) is similar to case (2).

**case** SUB-CTRANS, SUB-EXTENDS. We have  $A \leq C$  and  $C \leq D_1$  and  $A$  extends  $D_2$ . Applying the induction hypothesis to  $A \leq C$  and  $A \leq D_2$ , either (1)  $B = \text{Object}$  or (2)  $C \leq D_2$  or (3)  $D_2 \leq C$ . In case (2), applying the induction hypothesis to  $C \leq D_1$  and  $C \leq D_2$  yields the required result. In case (3), the result follows from transitivity of subclassing.

**case** SUB-EXTENDS, SUB-EXTENDS. Immediate from T-CLASS. □

**Lemma B.9.** If  $matchArg(m, D, \overline{M}) = M$ , then  $D \leq D'$  where  $arg_I(M) = D'$ .

*Proof.* By induction on  $matchArg$ .

**case** MATCH1.  $D = D'$ .

**case** MATCH2. We have  $matchArg(m, E_k, \overline{M}) = M$ , where  $D$  extends  $E_k$ . By the induction hypothesis,  $arg_I(M) = D'$  and  $E_k \leq E'_k$ . By SUBCTRANS,  $D \leq D'$ , which is the required result. □

**Lemma B.10.** If  $dispatch(m, C, D) = M$ , then  $D \leq D'$ , where  $arg_I(M) = D'$ .

*Proof.* Follows by induction on  $dispatch$  and Lemma B.9. □

**Lemma B.11.** If  $matchArg(m, D, \overline{M}) = M_k$ , for some unique  $M_k$  and  $\forall i. arg_I(M') \not\leq arg_I(M_i)$ , then  $matchArg(m, D, (\overline{M}, M'))$  has the unique result  $M_k$ .

*Proof.* By induction on  $match$ .



**case** MATCH1. Immediate from the fact that  $arg_I(M') \neq arg_I(M_k)$ .

**case** MATCH2. We have  $D$  extends  $\bar{E}_k$  and  $matchArg(m, E_k, \bar{M}) = M_k$ . By the induction hypothesis, there is a unique  $M_k$  where  $matchArg(m, E_k, (\bar{M}, M')) = M_k$ .

Suppose  $M' = B_0 m(\bar{D} \bar{x}, \bar{B} \bar{x})$ . Then, by MATCH1,  $matchArg(m, D, (\bar{M}, M')) = D$ . But, by Lemma B.9,  $E_k \leq arg_I(M_k)$  and therefore  $D \leq arg_I(M_k)$ . By premise,  $arg_I(M') \not\leq arg_I(M_k)$  so  $D \not\leq arg_I(M_k)$ , which is a contradiction.

Therefore,  $B_0 m(\bar{D} \bar{x}, \bar{B} \bar{x}) \notin (\bar{M}, M')$  and the rule MATCH2 applies, providing the required result. □

**Lemma B.12** (Weakening for matchArg). If  $matchArg(m, D, \bar{M}) = M_k$ , for some unique  $M_k$  and  $\forall i \in 1..\#\bar{M}'. \forall j \in 1..\#\bar{M}. arg_I(M'_i) \not\leq arg_I(M_j)$ , then  $matchArg(m, D, (\bar{M}, \bar{M}'))$  has the unique result  $M_k$ .

*Proof.* By induction on  $\bar{M}'$ .

**case**  $\bar{M}' = \bullet$ . Immediate.

**case**  $\bar{M}' = M_0, \bar{M}''$ . Result follows from Lemma B.11. □

**Lemma B.13.** If  $matchArg(m, A, \bar{M}) = M_0$  and  $A' \leq A$ , then  $matchArg(m, A', \bar{M}) = M'_0$  where  $arg_I(M'_0) \leq arg_I(M_0)$ .

*Proof.* By induction on  $matchArg(m, A, \bar{M})$ .

**case** MATCH1. Follows from Lemma B.7.

**case** MATCH2. We have  $A$  extends  $A_k$  and  $matchArg(m, A_k, \bar{M}) = M_0$ . By transitivity of subtyping,  $A' \leq A_k$ . The result then follows from the induction hypothesis. □

**Lemma B.14** (Sufficient conditions for *match* to be defined.). If we have  $\bar{M}$  where  $M_i = B_i m(A_i x, \bar{B}_i \bar{x}) \{ \text{return } e_i; \}$  and

1.  $D \leq A_\ell$  (for some  $\ell$ )
2.  $A_i \neq A_j$  (for all  $i \neq j$ )
3.  $D \leq A_i$  and  $D \leq A_j$ , implies  $A_i \leq A_j$  or  $A_j \leq A_i$  (for all  $i \neq j$ )

then there exists a unique  $M_k$  such that  $matchArg(m, D, \bar{M}) = M_k$  and for all  $j$  such that  $D \leq A_j$ ,  $A_k \leq A_j$ .

*Proof.* By induction on  $\bar{M}$ .

**case**  $\overline{M} = M_0$ , where  $arg_I(M_0) = D'$  and  $D \leq D'$ . The result follows from Lemma B.7.

**case**  $\overline{M} = M_0, \overline{M}'$ , where  $arg_I(M_0) = D'$ .

Either  $\exists k \in 1..#\overline{M}'. D \leq A_k$  (where  $arg_I(M_k) = A_k$ ), or not. If such a  $A_k$  does not exist (i.e.,  $\forall i. D \not\leq A_i$ ), by assumption  $D \leq D'$ . By Lemma B.6,  $matchArg(m, D, M_0) = M_0$ . From this it follows that  $\forall i. D' \not\leq A_i$ , and the result follows from Lemma B.12.

Otherwise, either  $D \leq D'$  or not. If  $D \not\leq D'$ , the result follows from Lemma B.12.

Therefore, we have  $D \leq D'$  and by assumption, for all  $i \in 1..#\overline{M}'$ ,  $D' \neq A_i$  and if  $D \leq A_i$ , either  $D' \leq A_i$  or  $A_i \leq D'$ , where  $A_i = arg_I(M'_i)$ .

By the induction hypothesis, there exists a unique  $M'_k$  such that  $matchArg(m, D, \overline{M}') = M'_k$  and  $\forall j. D \leq A_j$  implies  $A_k \leq A_j$ . Since by Lemma B.9,  $D \leq A_k$ , from above, either (1)  $D' \leq A_k$  or (2)  $A_k \leq D'$ .

In either case (1) or (2), by the induction hypothesis,  $matchArg(m, D, (M_0, M'_k)) = M''$ , where  $arg_I(M'') \leq D'$  and  $arg_I(M'') \leq A_k$ . By Lemma B.6,  $M'' \in \{M_0, M'_k\}$ .

In case (1), we can conclude that  $M'' = M_0$ . From above, we have  $\forall j. D \leq A_j$ ,  $D' \leq A_j$ . The result then follows from Lemma B.12.

In case (2), we can conclude that  $M'' = M'_k$ . The result then follows from Lemma B.12.

□

**Lemma B.15.** If class  $C \cdots \{ \overline{C} \overline{f}; K \overline{M} \}$  and  $M_1 \in \overline{M}$  where  $arg_I(M_1) = A_1$  and  $M_2 \in \overline{M}$  where  $arg_I(M_2) = A_2$  and  $A_1 \leq A_2$  and  $A \leq A_1, A_2$  and  $matchArg(m, A, \overline{M}) = M'$ , then  $arg_I(M') \leq A_1$ .

*Proof.* By case analysis on  $matchArg(m, A, \overline{M})$ .

**case** MATCH1. Immediate.

**case** MATCH2. We have  $A$  extends  $A_k$  and  $matchArg(m, A_k, \overline{M}) = M'$  and  $arg_I(M') = A'_k$ . By Lemma B.6,  $matchArg(m, A_1, \overline{M}) = M_1$ . Since the result of  $matchArg(m, A_k, \overline{M})$  is unique,  $A_k \leq A_1$ . Then, by Lemma B.13,  $A'_k \leq A_1$ .

□

**Lemma B.16.** If class  $C \cdots \{ \overline{C} \overline{f}; K \overline{M} \}$  and  $matchArg(m, A, \overline{M}) = M_1$  and  $matchArg(m, A, \overline{M}) = M_2$  then  $M_1 = M_2$ .

*Proof.* By Lemma B.6,  $M_1 = B m(\langle B' @ \rangle B_0 x, \overline{B} \overline{x}) \in \overline{M}$  and  $M_2 = B' m(\langle B'' @ \rangle B'_0 x, \overline{B}' \overline{x}) \in \overline{M}$ . By Lemma B.9,  $A \leq B_0$  and  $A \leq B'_0$ . There are 3 cases to consider: (1)  $\exists B'$  and  $\exists B''$ , (2)  $(\exists B'$  and  $\exists B''$ ) or  $(\exists B'$  and  $\exists B''')$ , or (3)  $\exists B'$  and  $\exists B''$ .

In case (1), by T-MULTI-METHOD,  $B_0 \leq B'$  and  $B'_0 \leq B''$ . Also by T-MULTI-METHOD,  $mtype(m, C) = (B', \overline{B}) \rightarrow B$  and  $mtype(m, C) = (B'', \overline{B}') \rightarrow B'$ . By the uniqueness of  $mtype$

(Lemma B.3),  $B' = B''$ . By T-MULTI-METHOD,  $B' \neq \text{Object}$ . But, since we have  $A \leq B_0$  and  $A \leq B'_0$ ,  $B_0 = B'_0$ ; otherwise this would violate the no-diamond property (Lemma B.8).

In case (2), suppose  $\exists B'$  and  $\nexists B''$  (the other case is analogous). By T-MULTI-METHOD,  $B_0 \leq B'$  and  $B_0 \neq B'$  and  $mtype(m, C) = (B_0, \bar{B}) \rightarrow B$ . By MTYPE1,  $mtype(m, C) = (B'_0, \bar{B}') \rightarrow B'$ . By the uniqueness of  $mtype$  (Lemma B.3),  $B'_0 = B'$ . We have  $A \leq B'$ ,  $A \leq B_0$  and  $B_0 \leq B'$ . By Lemma B.15,  $B' \leq B_0$ , which implies  $B_0 = B'$ . This is a contradiction.

In case (3), since there cannot be two unspecialized methods with the same name defined in  $C$ ,  $M_1 = M_2$ . □

**Lemma B.17.** If class  $C \cdots \{ \bar{C} \ \bar{f}; K \ \bar{M} \}$  and  $B \ m(A \ x, \bar{B} \ \bar{x}) \in \bar{M}$  and  $A' \leq A$ , then  $matchArg(m, A', \bar{M}) = M_0$  where  $arg_I(M_0) = A''$  and  $A'' \leq A$ .

*Proof.* By induction on  $A' \leq A$ .

**case** SUB-CREFL. Result follows from MATCH1.

**case** SUB-CTRANS. We have  $A' \leq A_1$  and  $A_1 \leq A$ . By the induction hypothesis,  $matchArg(m, A_1, \bar{M}) = A''$  where  $A'' \leq A$ . The result then follows from Lemma B.13.

**case** SUB-EXTENDS. We have  $A'$  extends  $A$ . Suppose  $B' \ m(A' \ x, \bar{B}' \ \bar{x}) \in \bar{M}$ . By MATCH1,  $arg_I(matchArg(m, A', \bar{M})) = A'$ , which gives the required result. Otherwise, if  $A'$  extends  $\bar{A}'$ , we must show that there exist unique  $k, M'$  such that  $matchArg(m, A'_k, \bar{M})$  is defined; MATCH2 then applies. By MATCH1, we have  $matchArg(m, A, \bar{M}) = M'$  and  $arg_I(M') = A$ .

Suppose  $\exists A'_j. matchArg(m, A'_j, \bar{M}) = M''$  and  $arg_I(M'') = B$ . By Lemma B.6,  $C \ m(\langle B_0 @ \rangle B \ x, \bar{B}' \ \bar{x}) \in \bar{M}$ . By Lemma B.9,  $A'_j \leq B$  so therefore  $A' \leq B$ . By T-METHOD,  $\exists D \neq \text{Object}. A \leq D$  and  $B \leq D$ . However, this means that a diamond results, which is impossible (Lemma B.8). Therefore, there exists a unique  $k$  where  $matchArg(m, A'_k, \bar{M}) = M_1$ . By Lemma B.16, this value is unique. □

**Lemma B.18.** If  $dispatch(m, C, A) = M$  and  $arg_I(M) = A'$  then  $mtype(m, C) = (A'', \bar{B}) \rightarrow B$  where  $A' \leq A''$ .

*Proof.* By induction on  $dispatch$ .

**case** DISPATCH1. We have class  $\cdots \{ \bar{C} \ \bar{f}; K \ \bar{M} \}$ . By Lemma B.6,  $B \ m(\langle B' @ \rangle A' \ x, \bar{B} \ \bar{x}) \in \bar{M}$ . Either (1)  $x$  has type  $B' @ A'$  or it has type  $A'$ . In case (1), by T-MULTI-METHOD,  $mtype(m, C) = (B', \bar{B}) \rightarrow B$ , where  $A' \leq B'$ . In case (2), the result follows from MTYPE1.

**case** DISPATCH2. We have  $dispatch(m, D, A) = M$ ,  $arg(M) = A'$  and  $C$  extends  $D$ . By the induction hypothesis,  $mtype(m, D) = (A'', \bar{B}) \rightarrow B$ , where  $A' \leq A''$ . The result then follows from MTYPE2.

□

**Lemma B.19.** If  $methodDef(m, C, A)$ , then  $mtype(m, C) = (B_0, \overline{B}) \rightarrow B$ , where  $A \leq B_0$ .

*Proof.* By induction on  $methodDef(m, C, A)$ .

**case** METHODDEF1. class  $C \cdots \{ \overline{C} \overline{f}; K \overline{M} \}$

There are two possible cases:

(1)  $B \ m(A \ x, \overline{B}) \in \overline{M}$ . Result follows from MTYPE1.

(2)  $B \ m(B' @ A \ x, \overline{B}) \in \overline{M}$ . By T-MULTI-METHOD,  $A \leq B'$  and  $mtype(m, C) = (B', \overline{B}) \rightarrow B$ , which is the required result.

**case** METHODDEF2. By the induction hypothesis,  $mtype(m, D_k) = (B_0, \overline{B}) \rightarrow B$ , where  $A \leq B_0$  and  $C$  extends  $D_k$ . The result then follows from MTYPE2.

□

**Lemma B.20.** If  $matchArg(m, D, \overline{M}) = M_0$ , and  $arg_I(M_0) = B'$  then  $methodDef(m, C, B')$ .

*Proof.* By induction on  $matchArg(m, D, \overline{M})$ .

**case** MATCHARG1. Result follows from METHODDEF1.

**case** MATCHARG2. Result follows from induction hypothesis.

□

**Lemma B.21.** If  $dispatch(m, C, D) = M_0$  and  $arg_I(M_0) = B'$ , then  $methodDef(m, C, B')$ .

*Proof.* By induction on  $dispatch(m, C, D)$ .

**case** DISPATCH1. We have  $matchArg(m, D, \overline{M}) = B_0 \ m(B' \ x, \overline{B} \ \overline{x})$ , where class  $C \cdots \{ \overline{C} \overline{f}; K \overline{M} \}$ . The result then follows from Lemma B.20.

**case** DISPATCH2. We have  $\overline{M}_E = \{ M_i \mid dispatch(m, E_i, D) \}$  and  $matchArg(m, D, \overline{M}_E) = B_0 \ m(B' \ x, \overline{B} \ \overline{x})$ , where  $C$  extends  $\overline{E}$ . By the induction hypothesis,  $methodDef(m, E_i, B') = E'_i$ , for some  $E'_i$  where  $E_i \leq E'_i$ . The result then follows from METHODDEF2 and the transitivity of subclassing.

□

**Lemma B.22.** If  $mtype(m, C) = (\text{Object}, \overline{B}) \rightarrow B$  and  $methodDef(m, C, A)$ , then  $A = \text{Object}$ .

*Proof.* By induction on the  $methodDef$  derivation.

**case** METHODDEF1. class  $C \cdots \{ \bar{C} \bar{f}; K \bar{M} \}$

There are two possible cases:

(1)  $B \ m(A \ x, \bar{B}) \in \bar{M}$ . By inversion on *mtype* and the uniqueness of *mtype* (Lemma B.3),  $A = \text{Object}$ .

(2)  $B \ m(B' @ A \ x, \bar{B}) \in \bar{M}$ . By the uniqueness of *mtype* (Lemma B.3), and premise (6) of T-MULTI-METHOD,  $B' = \text{Object}$ . But, by premise (3),  $B' \neq \text{Object}$ , so this case is impossible.

**case** METHODDEF2. We have *methodDef*( $m, D_k, A$ ), where  $C$  extends  $D_k$ . By Lemma B.19, *mtype*( $m, D_k$ ) =  $(B_0, \bar{B}') \rightarrow B'$ . By the uniqueness of *mtype* (Lemma B.3),  $\bar{B}' = \bar{B}$ ,  $B = B'$ , and  $B_0 = \text{Object}$ . The result then follows from the induction hypothesis. □

**Lemma B.23.** If we have the following:

1. class  $C \{ \bar{C} \bar{f}; K \bar{M} \}$  extends  $D_1, D_2$
2.  $m \notin \bar{M}$
3. *dispatch*( $m, D_1, E$ ) =  $M_1$  and *dispatch*( $m, D_2, E$ ) =  $M_2$
4. *arg<sub>1</sub>*( $M_1$ ) =  $A_1$  and *arg<sub>1</sub>*( $M_2$ ) =  $A_2$

then  $A_1 \neq A_2$  and  $(A_1 \leq A_2 \text{ or } A_2 \leq A_1)$ .

*Proof.* By Lemma B.10,  $E \leq A_1$  and  $E \leq A_2$ . By Lemma B.18, *mtype*( $m, D_1$ ) =  $(B_0, \bar{B}) \rightarrow B$  and *mtype*( $m, D_2$ ) =  $(B'_0, \bar{B}') \rightarrow B'$  where  $A_1 \leq B_0$  and  $A_2 \leq B'_0$ . By Lemma B.3,  $B_0 = B'_0$ , so  $A_1, A_2 \leq B_0$ .

By Lemma B.21, we have *methodDef*( $m, D_1, A_1$ ) and *methodDef*( $m, D_2, A_2$ ). Suppose  $A_1 = A_2$ . By T-CLASS,  $C' \ m(\langle A' @ \rangle C_0 \ x, \bar{C} \ \bar{x}) \in \bar{M}$ , which is a contradiction.

We can use the no-diamond property to prove the second conjunct of the result, once we have shown that  $B_0 \neq \text{Object}$ . By Lemma B.21, *methodDef*( $m, D_1, A_1$ ) and *methodDef*( $m, D_2, A_2$ ). Then *mtype*( $m, D_1$ ) =  $(\text{Object}, \bar{B}) \rightarrow B$  and *mtype*( $m, D_2$ ) =  $(\text{Object}, \bar{B}') \rightarrow B'$ . By Lemma B.22,  $A_1 = A_2 = \text{Object}$ , which contradicts the result proved above that  $A_1 \neq A_2$ .

Finally, by Lemma B.8, since  $B_0 \neq \text{Object}$ , either  $A_1 \leq A_2$  or  $A_2 \leq A_1$ , which completes the proof. □

**Lemma B.24.** If  $\mathcal{D} :: \text{mtype}(m, C) = (B_0, \bar{B}) \rightarrow B$  and  $\mathcal{D}$  does not contain the rule MTYPE3, and  $B' \leq B_0$ , then *dispatch*( $m, C, B'$ ) =  $M$  where *arg<sub>1</sub>*( $M$ ) =  $B''$  and  $B'' \leq B_0$ .

*Proof.* By induction on  $\mathcal{D}$ .

**case** MTYPE1. We have class  $C \cdots \{ \bar{C} \bar{f}; K \bar{M} \}$  and  $B \ m(B_0 \ x, \bar{B} \ \bar{x}) \in \bar{M}$ . By Lemma B.17, *matchArg*( $m, B', \bar{M}$ ) =  $M_1$  where *arg<sub>1</sub>*( $M_1$ ) =  $B''$  and  $B'' \leq B_0$ . The result then follows from DISPATCH1.

**case** MTYPE2. class  $C$  extends  $\bar{D} \{ \bar{C} \bar{f}; K \bar{M} \}$   $m \notin \bar{M}$   $\mathcal{D}_k :: \text{mtype}(m, D_k) = (B_0, \bar{B}) \rightarrow B$   
 By the induction hypothesis, *dispatch*( $m, D_k, B'$ ) =  $M_k$ , where *arg<sub>1</sub>*( $M_k$ ) =  $A_k$  and  $A_k \leq B_0$ .  
 Let  $\bar{M}_D = \{ M_i \mid \text{dispatch}(m, D_i, B') \}$ . It suffices to show that  $\exists$  unique  $M''$ . *matchArg*( $m, B', \bar{M}_D$ ) =  $M''$ ; the rule DISPATCH2 then applies.

Let  $A_i = \text{arg}_i(M_i)$ . By Lemma B.23, for all  $i \neq j$ ,  $A_i \neq A_j$  and either  $A_i \leq A_j$  or  $A_j \leq A_i$ . The result then follows from Lemma B.14. □

**Lemma B.25.** If  $\mathcal{D} :: \text{mtype}(m, D) = \overline{B} \rightarrow B$  and  $C <: D$  and  $\Gamma \vdash \text{new } C(\overline{e}) : C$ , then there exist  $D'$  and  $\mathcal{D}'$  such that  $C \leq D'$  and  $\mathcal{D}' :: \text{mtype}(m, D') = \overline{B} \rightarrow B$ , where  $\mathcal{D}'$  does not contain the rule MTYP3.

*Proof.* By induction on the *mtype* derivation.

**case** MTYP1. We observe that  $\mathcal{D}$  does not contain the rule MTYP4. There are two possibilities: either  $C \leq D$ , in which case let  $\mathcal{D}' = \mathcal{D}$ , or  $C \not\leq D$ . In the latter case, by Lemma B.5,  $\exists E \leq D$ .  $C$  requires  $E$ . But, this is impossible; by inversion on T-NEW,  $C$  requires  $\bullet$ .

**case** MTYP2. We have  $D$  extends  $D_k$  where  $\mathcal{D}_k :: \text{mtype}(m, D_k) = \overline{B} \rightarrow B$ . The result then follows from the induction hypothesis and MTYP2.

**case** MTYP3. Similar to above. □

**Lemma B.26.** If  $C \leq D$  and  $\mathcal{D} :: \text{mtype}(m, D) = \overline{B} \rightarrow B$  does not contain the rule MTYP3, then there exists  $\mathcal{D}' :: \text{mtype}(m, C) = \overline{B} \rightarrow B$  that does not contain the rule MTYP3.

*Proof.* Straightforward induction on  $C \leq D$ . □

**Lemma B.27.** If  $\text{mtype}(m, C) = (B_0, \overline{B}) \rightarrow B$  and  $\Gamma \vdash \text{new } C(\overline{e}) : C$ , and  $B' \leq B_0$ , then  $\text{mbody}(m, C, B') = \overline{x}.e_0$ , for some  $\overline{x}$  and  $e_0$ .

*Proof.* By case analysis on the derivation of *mtype*.

**case** MTYP1. By Lemma B.17,  $\text{matchArg}(m, B', \overline{M})$  is defined, where  $\overline{M}$  are the methods of  $C$ . By DISPATCH1,  $\text{dispatch}(m, C, B')$  is defined, which implies that  $\text{mbody}(m, C, B')$  is also defined.

**case** MTYP2. We have  $\text{mtype}(m, D_k) = (B_0, \overline{B}) \rightarrow B$ , where  $C$  extends  $D_k$ . By Lemmas B.25 and B.26, there exists  $\mathcal{D} :: \text{mtype}(m, C) = (B_0, \overline{B}) \rightarrow B$  that does not contain rule MTYP3. By Lemma B.24,  $\text{dispatch}(m, C, B')$  is defined, which implies that  $\text{mbody}(m, C, B')$  is also defined.

**case** MTYP3. Vacuous; by inversion of T-NEW,  $C$  requires  $\bullet$ . □

**Theorem B.1** (Progress). If  $\cdot \vdash e : C$  then either  $e$  is a value or there is an  $e'$  with  $e \mapsto e'$ .

*Proof.* By induction on  $e : C$ .

**case** T-VAR. Vacuous.

**case** T-FIELD.  $e = e_0.f_i$

We have  $fields(C_0) = \overline{C}.f$ . By the induction hypothesis, either  $e_0$  is a value or it evaluates to some  $e'_0$ . In the first case, the rule T-FIELD1 applies. In the second case, the rule E-FIELD2 applies.

**case** T-INVK.  $e = e_0.m(\overline{e}) \quad \overline{e} : \overline{C} \quad \overline{C} <: \overline{D}$

By the induction hypothesis, either  $e_0$  is a value or it evaluates to some  $e'_0$ . If it evaluates, then the rule E-INVK-RECV applies. If it is a value, then either the arguments  $\overline{e}$  evaluate or they are values. In the first case, E-INVK-ARG applies.

Otherwise, by assumption,  $mtype(m, C_0) = \overline{D} \rightarrow D$  and  $e_0 : C_0$  and  $e_1 : C_1$  where  $C_1 <: D_1$ . By inversion on T-NEW, we have  $e_0 = \text{new } C_0(\overline{e}'_0)$  and  $e_1 = \text{new } C_1(\overline{e}'_1)$  and  $C_1$  requires  $\bullet$ . By Lemma B.4,  $C_1 \leq D_1$ . By Lemma B.27,  $mbody(m, C_0, C_1)$  is defined; the rule E-INVK then applies.

**case** T-SUPER-INVK.  $e = e_0.D.super.m(\overline{e})$

By the induction hypothesis, either  $e_0$  is a value or it evaluates to some  $e'_0$ . If it evaluates, then the rule E-INVK-SUPER-RECV applies. If it is a value, then either the arguments  $\overline{e}$  evaluate or they are values. In the first case, E-SUPER-INVK-ARG applies.

Otherwise, by inversion on T-NEW we have  $e_0 = \text{new } C_0(\overline{e})$  and  $e_1 = \text{new } C_1(\overline{e}'_1)$  and  $C_1$  requires  $\bullet$ . By assumption,  $mtype(m, D) = \overline{D} \rightarrow C$ . Since  $C_1 <: D_1$  by Lemma B.4,  $C_1 \leq D_1$ . Let  $E = \text{super}(C, D)$ . By the definition of *super*, we have  $E \leq D$ . By Lemma B.1,  $mtype(m, E) = \overline{D} \rightarrow C$ . By Lemma B.27,  $mbody(m, E, C_1)$  is defined. The rule E-SUPER-INVK then applies.

**case** T-NEW.  $e = \text{new } C(\overline{e})$

By the induction hypothesis, either  $\overline{e}$  evaluates or it is a value. If it evaluates, the rule E-NEW-ARG applies. Otherwise, the expression itself is a value.

□

### B.3 Preservation Lemmas and Proof

**Lemma B.28** (Substitution). If  $\Gamma, \overline{x} : \overline{C} \vdash e : D$  and  $\Gamma \vdash \overline{d} : \overline{C}'$  where  $\overline{C}' <: \overline{C}$  then  $\Gamma \vdash [\overline{d}/\overline{x}] e : D'$  for some  $D' <: D$ .

*Proof.* Similar to proof of FJ, using Lemma B.2 for the case of method invocation. □

**Lemma B.29** (Weakening). If  $\Gamma, x : C, \Gamma' \vdash e : B$  then for  $C' <: C$  and  $B' <: B$ ,  $\Gamma, x : C', \Gamma' \vdash e : B'$ .

*Proof.* Straightforward induction on typing derivations. □

**Lemma B.30.** If  $\text{dispatch}(m, C, D) = \overline{x}.e_0$  then there exists a unique  $\overline{B} \rightarrow B$  such that  $mtype(m, C) = \overline{B} \rightarrow B$ .

*Proof.* By induction on the derivation of *dispatch*.

**case** DISPATCH1. By Lemma B.6,  $m \in \overline{M}$ . The result then follows from MTYPE1.

**case** DISPATCH2. By Lemma B.6,  $M'' \in \overline{M}_E$ . By definition,  $M'' = \text{dispatch}(m, E_k, D)$  where  $C$  extends  $E_k$ . By the induction hypothesis,  $\text{mtype}(m, E_k) = \overline{B} \rightarrow B$ . By MTYPE2,  $\text{mtype}(m, C) = \overline{B} \rightarrow B$  and by Lemma B.3, this value is unique.

□

**Lemma B.31.** If  $\text{dispatch}(m, C, D) = M'$  where  $M' = E \ m(E_0 \ x, \overline{E} \ \overline{x}) \ \{\text{return } e;\}$  and  $\text{mtype}(m, C) = (B_0, \overline{B}) \rightarrow B$  then there exists some  $B' <: B$  such that  $x : B_0, \overline{x} : \overline{B}, \text{this} : C \vdash e : B'$ .

*Proof.* By induction on the definition of *dispatch*( $m, C, D$ ).

**case** DISPATCH1. By Lemma B.6,  $M' = D_0 \ m(\langle D \rangle D' \ x, \overline{D} \ \overline{x})$  and  $M' \in \overline{M}$ , where  $D' \leq D$ . By inversion on T-METHOD and T-MULTI-METHOD, the result follows.

**case** DISPATCH2. By Lemma B.6,  $M' \in \overline{M}_E$ . By definition,  $M' = \text{dispatch}(m, E_k, D)$  where  $C$  extends  $E_k$ . By Lemma B.30,  $\text{mtype}(m, E_k) = \overline{C} \rightarrow C'$ , for some unique  $\overline{C} \rightarrow C'$ . Since  $\nexists M'. \text{matchArg}(m, D, \overline{M}) = M'$ , by Lemma B.7,  $C_0 \ m(\langle C \rangle D' \ x, \overline{C} \ \overline{x}) \notin M'$ , where  $D \leq D'$ . So, by MTYPE2,  $\text{mtype}(m, C) = \text{mtype}(m, E_k)$ . Since the result of *mtype* is unique (Lemma B.3), we have  $\overline{B} = \overline{C}$  and  $B = C'$ . Applying the induction hypothesis to *dispatch*( $m, E_k, D$ ) and *mtype*( $m, E_k$ ) yields the required result.

□

**Theorem B.2** (Preservation). If  $\Gamma \vdash e : C$  and  $e \mapsto e'$ , then  $\Gamma \vdash e' : C'$  for some  $C' <: C$ .

*Proof.* By induction on derivation of  $e \mapsto e'$ .

**case** E-FIELD.

$$\begin{aligned} e &= (\text{new } C_0(\overline{e})).f_i \\ e' &= e_i \\ \text{fields}(C_0) &= \overline{D} \ \overline{f} \\ D_i &= C \end{aligned}$$

By the rule T-FIELD,  $\Gamma \vdash \text{new } C_0(\overline{e}) : C_0 \quad C_0 <: C_0$ .

By T-NEW,  $\Gamma \vdash \overline{e} : \overline{C} \quad \overline{C} <: \overline{D} \quad C_0 = C_0$ .

By transitivity of subtyping,  $e_i : D_i$ , which is the required result.

**case** E-INVK.

$$\begin{aligned} e &= (\text{new } C_0(\overline{e})).m(\text{new } D(\overline{e}'), \overline{d}) \\ e' &= [\text{new } D(\overline{e}')/x, \overline{d}/\overline{x}, \text{new } C_0(\overline{e})/\text{this}] e_0 \\ \text{mbody}(m, C_0, D) &= (x, \overline{x}).e_0 \end{aligned}$$

By T-INVK and T-NEW:

$$\Gamma \vdash \text{new } C_0(\overline{e}) : C_0$$



$$\begin{aligned}
&\Gamma \vdash \text{new } D(\bar{e}') : D \quad D <: D_0 \\
&\Gamma \vdash \bar{d} : \bar{B} \quad \bar{B} <: \bar{D} \\
&\text{mtype}(m, C_0) = (D_0, \bar{D}) \rightarrow C
\end{aligned}$$

From the definition of *mbody*, we have  $\text{dispatch}(m, C_0, D) = E \ m(E_0 \ x, \bar{E} \ \bar{x}) \ \{\text{return } e;\}$ . By Lemma B.31, there exists some  $C' <: C$  such that  $x : D_0, \bar{x} : \bar{D}, \text{this} : C_0 \vdash e_0 : C'$ . By Lemma B.28,  $\cdot \vdash [\text{new } D(\bar{e}')/x, \bar{d}/\bar{x}, \text{new } C_0(\bar{e})/\text{this}] e_0 : C''$ , for some  $C'' <: C'$ . By the transitivity of subtyping,  $C'' <: C$ , which is the required result.

**case** E-SUPER-INVK.

$$e = (\text{new } C_0(\bar{e})).B.\text{super}.m(\text{new } D(\bar{e}'), \bar{d})$$

By T-SUPER-INVK and T-NEW:

class  $C_0$  requires  $B, \bar{E}$   
class  $C_0$  requires •

This is a contradiction, therefore this case is vacuous (dynamically-dispatched super calls can only be applied to classes with a non-empty requires clause).

The cases for the congruence rules are straightforward.

□