The Knowledge-Learning-Instruction (KLI) Framework: Toward Bridging the Science-Practice Chasm to Enhance Robust Student Learning

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June 7, 2010
CMU-HCII-10-102

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This work is supported by the National Science Foundation under grant SBE0836012 and the Pittsburgh Science of Learning Center. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

Keywords: computational modeling, cognitive modeling, instructional theory, machine learning, learning science, second language learning, mathematics learning, science learning, robust learning, learning theory, knowledge components

Executive Summary

The volume of research on learning and instruction is enormous. Yet progress in improving educational outcomes has been slow at best. Many learning science results have not been translated into general practice and it appears that most that have been fielded have not yielded significant results in randomized control trials. Addressing the chasm between learning science and educational practice will require massive efforts from many constituencies, but one of these efforts is to develop a theoretical framework that permits a more systematic accumulation of the relevant research base.

A key piece in such a theoretical framework is the development of levels of analyses that are fine enough to be supported by cognitive science and cognitive neuroscience, but also at levels appropriate to guide the design of effective educational practices. An ideal scientific solution would be a small set of universal instructional principles that can be applied to produce efficient and robust student learning for any educational goal. This holy grail is likely unattainable both because effective instructional practices in one subject-matter domain, like science, are often not effective in another, like second language and because even within a domain, specific instructional goals and contexts add restrictions to the application of principles. Thus, our strategy is to strive for instructional principles with as much generality as possible, while recognizing that (and stipulating how) their effectiveness may be constrained by student and knowledge characteristics of the target course.

What are the features of knowledge that may so constrain? To answer that question, we have embarked on an effort to create taxonomy of kinds of knowledge with a focus on functional cognitive characteristics that may determine which learning processes are most likely to produce different kinds of knowledge. Cognitive science has identified a vast array of learning strategies, processes, and mechanisms and these are also in need of order. Similarly, the learning and educational sciences have produced an array of instructional principles and practice recommendations. Our theoretical framework builds on work in these fields. The Knowledge-Learning Instruction (KLI) Framework encompasses the three components that constitute its name. Although we focus in this paper on the knowledge taxonomy, we begin to outline taxonomies of learning processes and of instructional principles. We suggest "complexity" as a key organizing principle and the use of the time course of the application of a unit of knowledge, a learning process, or an instructional principle as operational method for empirically grounding this notion of complexity.

The purpose of the framework and the taxonomic efforts is not just collection and organization, but generation of new research questions and hypotheses. One hypothesis-generating frame involves the relationship between the complexity of components of knowledge targeted by a course and the complexity of the instructional principles that are likely to be most effective and efficient in enhancing robust student learning in that course. See Table 5. We hypothesize that an instructional principle at a particular level of complexity will be effective for knowledge components of similar complexity and those of greater complexity, but not for ones of less complexity. In other words, complex instructional strategies (intended to support complex learning processes involving deliberate reasoning and sense-making) are best used only for the most complex of knowledge component goals, but simpler instructional strategies (intended to support simpler learning processes like memory) are relevant and effective for knowledge

component goals of all complexity. A specific instance of this hypothesis is that the moderately complex instructional principle of prompting students to self-explain (or asking deep questions) is effective and efficient for learning more complex knowledge components (e.g., principles, like electric field properties, in mathematics and science) but is not effective and efficient for learning less complex knowledge components (e.g., categorical decisions, like English articles, in second language learning). The complexity dimension does not collapse onto domain differences. Thus, second language learning contains complex as well as simpler knowledge components, and so do math and science.

Thus the KLI framework has properties we think are essential to making a bridge from research to education. Its analysis of learning in terms of multi-level knowledge components reveals complexities that allow generalizations across domains. These generalizations, in turn, support instructional principles of high generality that, when combined with instructional goals, allow practical suggestions about curricula and intervention decisions.

1. Introduction: The need for a learning-to-instruction theoretical framework

The cognitive and learning sciences have a substantial base of highly refined and extensively tested theories of cognition and learning (e.g., Anderson & Lebiere, 1998; McClelland, Cleeremans, & Servan-Schreiber, 1990; Newell, 1990; Sun, 1994). For issues of educational science, specifically the application of a learning science to a science of instruction, such theories provide broad useful but limited guidelines. In designing instruction to support learning, one must deal with the complexities, tradeoffs, and uncertainties that arise in every transition from basic science to application. In crafting instruction – from a single response of a computer-based intelligent tutor to a teacher managing a classroom discussion to a full curriculum – the designer faces an enormous space of possible actions, in contexts where systematic, cleanly controlled, experimental contrasts of alternative approaches are impractical at best and often impossible. A theoretical framework that, in its very conception, included issues of instruction would be a significant aid to instructional applications of learning science research. Such a framework could help support "rigorous, sustained scientific research in education", as urged by the National Research Council report (Shavelson & Towne, 2002).

In the following sections, we first develop the basic background that motivates our attempt to formulate a general theoretical framework for enhancing student learning. We then define and explain the basic concepts that shape the theoretical framework, then provide elaboration and examples across academic domains, and identify exemplary instructional principles that derive from research using the theoretical framework. We conclude by suggesting how the framework can aid a sustained research effort and generate new hypotheses.

1.1. The Educational Context

Heated debates in education, sometimes dubbed "education wars', have been most visible in reading and math. Research has informed these debates, particularly in reading, but many of the core premises in these debates remain as assumptions with insufficient scientific support. Further, advocates in these debates tend to differ on the kind of scientific support that is needed, with some emphasizing rigor (e.g., internal validity, randomized controlled experiments) and others emphasize relevance (e.g., ecological and external validity and appropriate settings and assessments). Our approach embeds rigorous experimental research methods in the context of real learning, with real students, in real contexts.

1.2 Theories of Learning and Instruction

Our emphasis on connecting learning research to instructional principles is not unique. Research in the learning sciences has led to statements of principles, including the influential NRC report "How People Learn" (Bransford, Brown, & Cocking, 2000), instructional design principles (e.g., Clark & Mayer, 2003), and learning principles drawn from Psychology research (http://www.psyc.memphis.edu/learning/principles/; Graesser, 2009). Progress in the "reading wars" was marked by a succession of evidence-based recommendations that draw on principles of learning to read alphabetic writing systems (Bransford et al, 2000; Snow et al 1998, National Reading Panel, 2000). Broad learning principles also have been directed specifically at teachers and educational decision makers (e.g., ***http://www.instituteforlearning.org/***; Pashler et al., 2007). We join the tradition of extracting principles from research, because we believe that finding the right level to communicate evidence based-generalizations is important for guidance in instruction. However, we also are committed to the idea that any principle must be based on sound evidence that supports the *generality* implied by the principle or identifies its limiting conditions. Achieving generality requires evidence in real instructional contexts and across a range of content domains.

Finding the right level of analysis or grain size for theory is a major question. Like all theories, including physical theories, the theoretical entities vary in the levels of analysis to which they apply, both in terms of the grain of the unit (micro to macro) and its functional level (neurological, cognitive, behavioral, social). We refer to these two together as "grain size". In the long run, we expect translation (not necessarily reduction) across adjacent levels, for instance, neurological to cognitive and vice versa. However, for coherence one must choose a grain-size and adopt it fairly consistently, with the realization that translation to larger and smaller grain sizes can occur.

Learning and education theories have tended to use macro levels of analysis with units at large grain sizes. For example, situated learning, despite its origins as a general proposition about the social-contextual basis of learning (Lave and Wenger, 1991), has through its extension by others (e.g., Greeno, 1998), become an educational hypothesis. As a scientific idea, situated learning has merit and is subject to empirical investigation. However it tends to use a rather large grain size including groups and environmental features as causal factors in performance. It also tends to focus on rich descriptions of cases studies and not on claims that may generalize beyond these cases. These features work against two important goals that we have: (1) Identifying mechanisms of student learning that lead to instructional principles. (2) Communicating instructional principles that are general over contexts and meaningful to instructional design. We assume that learning is indeed situated, and understanding learning environments, including social interaction patterns that support learning, is an important element of a general theory. But we take the interaction between a learner and a specific instructional environment as the unit of analysis.

Learning theories have small grain sizes as well. At the neural level, variations on Hebb's idea (Hebb, 1949) about neuronal plasticity refer to a basic mechanism of learning, often expressed simplistically as "neurons that fire together, wire together". Neuron network models built this way have provided neurological correlates of behavioral learning and models for cognitive learning mechanisms using multi-level networks (e.g., O'Reilly & Munakata, 2000). Micro-level theories also can provide accounts of elementary causal events by using symbols, rules, and operations on basic entities expressed as computer code, as embodied, for instance, in the ACT-R theory of Anderson (1990; 1993). Although such theories can be and were developed without attention to biology, their predictions have been linked to brain systems (e.g., Anderson et al., 2004). It is important for the learning sciences that such theories, and their innovative predecessors (e.g., Newell & Simon, 1972), demonstrate the ability to predict and explain human cognition at a detailed level that is subject to empirical testing. The small grain size of such theories and the studies that support them leave them mostly untested at the larger grain size of knowledge-rich academic learning and they tend to be insufficient to constrain instructional design choices. There are exceptions, however. For instance, detailed theories about learning to read have been used to specify effective instructional interventions that affect brain circuits for learning to read (Sandak et al, 2004; Shaywitz et al, 2004). Moreover, without this level of explanation, we would have little confidence that the levels of explanation that suits our purpose could ever be translated to a precise mechanistic account. Our point is that to directly target instructional decisions that we believe should be influenced by learning science, an intermediate level is needed.

The levels of explanation we target are indeed intermediate to the larger and smaller grain sizes exemplified above. We believe that the aim of communicating the essential ideas of an instructionally relevant science of learning requires an intermediate level. This level must contain propositions whose scope is observable entities related to instructional environments and learner characteristics that affect learning events. These propositions are empirical claims, so they are testable by experiments. Finally, the learning propositions allow translation both downward to micro-level mechanisms and upward to instructional principles. The theoretical framework we describe in section 2 uses this intermediate cognitive level, which we refer to as the *learning event level.* First, we describe a research context for theory development.

1.3 Situating the theoretical framework in a student learning context

The Pittsburgh Science of Learning Center (PSLC) studies student learning of academic content as taught in courses. We are developing a theoretical framework that, because it is grounded in real student learning situations, applies directly to course-based learning. Important to this approach is that the learning situations span a range of academic content.

The advantage of studying multiple academic content areas is that generalizations and limits on generalizations become visible, allowing theoretical propositions and instructional principles to be expressed with appropriate boundary conditions. We do not assume that learning a second language and learning physics are identical processes at all levels of analysis. Instead, we assume that studies of learning are intrinsically about learning domain content and the scientific goal is to develop both general theoretical propositions and instructional principles across domains and to identify the domain-specific constraints on learning.

The PSLC research program has spanned the following student learning domains, each an academic course situated in one or more K12 Schools or University settings that cooperate with the PSLC research program: Physics, Chemistry, Algebra, Geometry, French, Chinese, and English as a second language. These academic courses have been studied at the early college, high school, or middle school level, a fact that imposes a potential grade- or experience-level boundary condition for generalizations based on the research. However, every theoretical proposition and instructional principle derived from this research is an empirically testable hypothesis for other grade or experience levels, like elementary school or adult job training.

1.4. Research Settings: In vivo experiments, lab experiments, and data logging

The goal of grounding our theoretical claims in research settings that represent application settings is best met, in our case, by research within operating courses and with students participating in those courses without any extra incentives. Course-based studies of all kinds are common in educational research, and they vary along many dimensions: whole classroom or individual student changes; in the classroom or "student pull-out"; domain-derived tasks or not; rigorous control or not; etc. To build the kind of learning theory to which we aspire, some classroom settings and research procedures are privileged, whereas a wider range of settings is important and necessary. The essential standards are real students, real subject matter, and rigorous control. Learning research has tended to trade-off laboratory rigor against classroom realism, with the former sometimes limited in generalization to classroom settings and the later sometimes suffering from lack of experimental rigor. We advocate research that combines the realistic settings of course-based research with experimental control that is as rigorous as possible within these real-world constraints. PSLC's primary methodology is called *in vivo* experimentation and it is described on the PSLC wiki (learnlab.org/research/wiki) and in other PSLC talks (e.g., Klahr, Perfetti, & Koedinger, 2009, VanLehn & Koedinger, 2007; Salden & Aleven, 2009; Salden & Koedinger, 2009) and publications (e.g., Koedinger et al., 2009).

Because *in vivo* experimentation comes with restrictions, including timing (i.e., the experiment runs as constrained by the course schedule) and ethical restrictions (e.g., including a condition hypothesized to be worse than current practice is not allowed), sometimes laboratory experimentation is preferred particularly at an earlier stage of development or to disentangle alternative explanations of results. However, these laboratory studies maintain important aspects of ecological validity by using the same course materials and same student population, while they lose the contextual and motivational backdrop of a study within a course.

Another source of empirical evidence for theory development is extensive logging of student interactions as they occur in experiments and in LearnLab courses, whether as part of an experiment or as part of normal course operation. Such data provide a source for further testing causal hypothesizes about why a particular instructional treatment is working or not. With the increasing use of educational data mining methods, such records become important resources for testing hypothesis about learning and providing data for testing simulations of learning models. The PSLC established its DataShop (learnlab.org/datashop; Koedinger, Baker, Cunningham, Skogsholm, & Stamper, in press) for exactly this purpose. It is a computer repository for experimental data obtained by classroom research that is open to researchers worldwide. As of June 2007, DataShop had over 160 data sets available for analysis, but of course that number continues to increase. The important point is that one can test hypotheses about student learning by use of such a database instead of doing an experiment that, in some aspects important to the researcher, has already been done.

Having set some conceptual and methodological context, we turn to a description of the theoretical framework.

2. The Knowledge-Learning-Instruction (KLI) Framework

The aim of the theoretical framework is to provide a general set of learning and instructional concepts that can be used to help explain student learning. It is a framework because it does not, at the level of theory, generate precise, falsifiable predictions. However, the framework does entail a hypothesis-testing research agenda. We use the framework versus theory distinction consistent with Anderson (1993, p. 2, with "cognition" replaced with "learning").

> "*Frameworks* are composed of the bold, general claims about [learning]. They are sets of constructs that define important aspects of [learning] … Frameworks, however, are insufficiently specified to enable predictions to be derived from them, but they can be elaborated, by the addition of assumptions, to make them into *theories*, and it is these theories that generate predictions. A single framework can be elaborated into many different theories."

We emphasize that the propositions within the framework can be used to generate hypotheses within specific domains and instructional situations that, with further work, lead to precise and falsifiable predictions. Indeed, we do desire that researchers will elaborate the framework to generate theoretical predictions. However, our main goal here is to identify broad claims and constructs on which many researches can agree, even those with different theoretical positions (e.g., symbolic versus connectionist theories, cognitive vs. social-situated theories).

2.1. Robust learning.

The scope of the framework is to give an account of the conditions that yield *robust student learning*: learning that lasts over time (long-term retention) and that transfers to new situations that differ from the learning situation along various dimensions (e.g., superficial differences in materials and assessment events). One kind of transfer seems special enough to warrant a category of its own, which we call accelerated future learning—learning in a new domain that can take advantage of prior learning in a domain (Bransford & Schwartz, 1999). This accelerated future learning can include the idea of learning how to learn, that is, acquiring general learning strategies that can apply to a wide range of situations, as well as conceptual learning in which concepts are learned deeply enough (i.e., in general enough form) to transfer to a new domain. For example, prompting students to self-explain in a Physics unit on electricity led to accelerated future learning in a magnetism unit (Hausmann & VanLehn, 2007). Such an effect can arise either because treatment students improved their learning strategies by becoming better self-explainers or because they learned electricity concepts (e.g., electrical field principles) in a deeper, more general way, such that it was easier to adapt those concepts to acquire similar magnetism concepts (e.g., magnetic field principles). Differences in prior experience can also lead to accelerated future learning in language as well. For example, a native English speaker who learns French as a second language will learn the specific system of French gender marking. This learning could provide a sensitivity to gender marking that would accelerate the learning of another second language that has gender marking, even if it had no immediate transfer effect on initial performance. Again, a study of such an effect needs to test whether it is about general "learning to learn" or specific concepts that can be applied to new situations.

2.2. Learning Events, Instructional Events, Assessment Events

Central to analysis with the KLI framework is a set of observable and unobservable events: *learning events*, *instructional events*, and *assessment events*, which decompose the temporal progression of learning. Figure 1 represents the relationships among the three events. Instructional and assessment events are observable changes to the instructional environment controlled by instructional designer or instructor. Instructional events are intended to produce learning (they evoke learning events). One can observe instructional events: a lesson, a 10 second episode on a computer, or a series of didactic moves by a teacher or computer tutor. Assessment events involve student response that is evaluated. Some assessment events are instructional, some are not. Test items are the most common kind of assessment event, but embedded assessments can occur in the context of instruction also, for instance, whether or not a student is correct without help on steps in a tutored problem-solving scenario. The unobservable entity in the triangle is the learning event, a central focus of the framework. Learning events are essentially changes in cognitive and brain states that can be inferred from data, but cannot be directly observed (brain imaging may be an exception) or directly controlled through experimental manipulation. The nature of these learning processes and knowledge changes are inferred from assessments of both immediate performance and robust performance, that is, performance that is remote in time (long-term retention) or in nature (transfer or future learning) from the activities during instruction. More detailed inferences about learning events can be informed experimentally, that is, by contrasts in performance due to differences in instructional or assessment events.

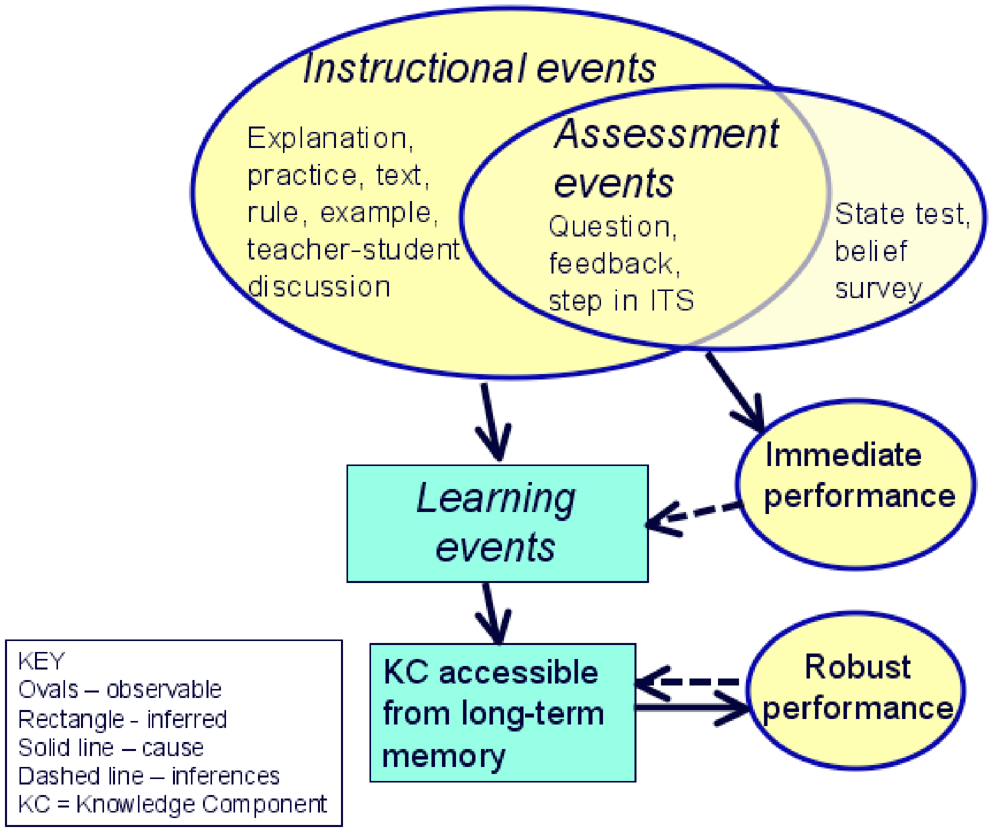

Figure 1. Understanding learning at the event level: Knowledge is acquired or modified during unobservable *learning events* inside students' minds. These learning events are influenced by *instructional events*, in which the student's learning environment is modified, and *assessment events*, in which student knowledge is inferred from performance either at the time of the event or later.

The goal of this event decomposition is to explain how observable instructional events affect unobservable learning events and associated changes in knowledge that can then be measured by assessment events, both in the near-term and long-term. There are scale issues to consider in this event analysis, especially time scale. Our focus is on intermediate size functional cognitive units and associated instructional, learning, and assessment events. Such events may vary in time from about 1 second, for instance, for a fact retrieval task with instructional feedback, to many minutes, for instance, a classroom dialogue around the concept of an integer.

To apply this framework, we decompose learning in two ways, temporally and structurally. Temporally, we can decompose the instructional and learning process over time into smaller segments, or *learning events*, which are unobservable and usually bracketed by some change in instructional input (an instructional event) or some request for student response (an assessment event). Structurally, we can decompose the otherwise undifferentiated changes occurring in the students' minds into *knowledge components*. Learning events produce knowledge components (or changes thereof) and acquisition of these components is the purported explanation for consistency (e.g., in terms of similar error rate, response latency, or explanation) in student performance and transfer from one assessment event to another. The next section presents the notion of knowledge components and a taxonomy of different kinds of knowledge components.

3. Knowledge: Decomposing Task Complexity and Transfer

Others have argued for the importance for educational design of analyzing learning goals into components of knowledge (e.g., diSessa, 1993; Minstrell, 2001). For instance, Anderson and Schunn (2000) suggested "there is a real value for an effort that takes a target domain, analyzes it into its underlying knowledge components, … communicates these components, and monitors their learning". Use of such cognitive task analysis to design instruction has been demonstrated to be quite effective in a number of training domains (e.g., Clark, Feldon, van Merriënboer, Yates, & Early, 2007; Lee, 2003), however, it is not a common approach for designing academic instruction. Cognitive task analysis remains as much an art as a science, in part, because of the unobservable nature of knowledge and limited scientific tools for characterizing it at a useful level of analysis. Thus, we think an effort toward defining a taxonomy of kinds of knowledge components is worthwhile and can be useful, we submit, even without costly implementation of computational models that has been the traditional approach of cognitive science.

We define a *knowledge component* is an acquired unit of cognitive function or structure that can be inferred from performance on a set of related tasks. As a practical matter, we use "knowledge component" broadly to generalize across (and potentially ignore important distinctions between) other terms for describing pieces of cognition or knowledge, including theoretical terms like schema (e.g., van Merriënboer & Sweller, 2005; Cheng & Holyoak, 1985), production rule (e.g., Newell & Simon, 1972; Anderson & Lebiere, 1998), misconception (e.g., Clement, 1987), or facet (e.g., Minstrell, 2001) as well as more common terms such as concept, principle, fact, or skill (c.f., Bloom, 1956). Knowledge components describe mental processes at about the ***unit task*** level within Newell's (1990) time scales of human action (see Table 1). Unit tasks last about 10 seconds (between 1 second and a minute) are at the interface between what Newell called the "cognitive band" and "rational band". Scientific investigation at these intermediate time scales (around a minute) is critical to make productive bridges between current neuroscience advances within biological band (where millisecond mental processes are being explored) and educational research within the social band (where learning processes that recur over months are being explored).

Table 1
Newell's Time Scales of Human Action

Scale (sec)	Time Units	System	World (theory)
10^7	months		
10^6	weeks		Social Band
10^5	days		
10^4	hours	Task	
10^3	10 min	Task	Rational Band
10^2	minutes	Task	
10^1	10 sec	Unit task	
10^0	1 sec	Operations	Cognitive Band
10^{-1}	100 msec	Deliberate act	
10^{-2}	10 msec	Neural circuit	
10^{-3}	1 msec	Neuron	Biological Band
10^{-4}	$100\mu s$	Organelle	

Table 2 provides some examples of kinds of knowledge components within second language learning, math, and science domains. Figure 2 shows sample learning curves of student performance in these domains. These curves illustrate, among other things, a rough correspondence between the complexity of a knowledge component and the average time for a correct application (the left column in Figure 2). As we illustrate below, there are many different kinds of knowledge components, some are simpler and faster to apply and others are more complex and slower to apply. Distinguishing kinds of knowledge components and their complexity is important not only because deep analysis of domain knowledge is a powerful source for innovative instructional design, but also because different kinds of learning processes and instructional treatments may be more or less effective depending on the nature and complexity of the target knowledge.

Knowledge components that function in a given task can be wide ranging, multi-leveled, overlapping, and hierarchical (or not), thus generally posing a problem of how to target the right components for a given learning situation. Because the KLI framework targets the analysis of academic learning, our strategy is to focus on knowledge that is to be acquired by students in the academic course under analysis. This general strategy leads to a more specific one, to focus on the component level at which the novice student makes errors. Thus, while components are often embedded in other components and hierarchical (e.g., sentence comprehension relies on word identification, which relies on letter recognition), we choose the levels that are associated with the skill level and error performance that is actually observed. For second graders, word reading errors may lead to a focus on knowledge components relevant for word identification that are above the grapheme (letter) level. For adults learning Chinese, word reading errors could lead to a focus on the grapheme (character) level, stroke level, or radical level, depending on observable error patterns.

Table 2. Examples of different kinds of knowledge components in Second Language Learning, Mathematics and Science

Domain / Knowledge Category	Application Conditions	Response	Example
2nd Language			
Chinese vocabulary	Constant	Constant	Chinese radical “日” => “sun” in English
English determiners	Variable	Constant	Choose article for <Noun> & it is definite => use “the”
Plurals	Variable	Variable	<Noun> ending <cons> “y” => remove “y” from <Noun> & add “ies”
Mathematics			
Rule from name	Constant	Constant	“circle area formula” = > “$A=\pi*r^2$”
Rule from context	Variable	Constant	Find area of <Fig> & it is a circle => use “$A=\pi*r^2$”
Solution planning	Variable	Variable	Find area of irregular shape made up of regular shapes <S1> and <S2> => find area <S1> and <S2> and add
Science			
Content fact	Constant	Constant	“gravity constant” => “$g = 9.8/s^2$”
Categorize situation	Variable	Constant	Find unit for <quantity>, a small volume => use “milliliters”
Discovery process	Variable	Variable	Test hypothesis <Var1> causes <Var2> => run an experiment that varies only <Var1> and nothing else

3.1 Kinds of knowledge components

We describe examples of knowledge components (KCs) starting with simple kinds with a shorter time scale and move toward increasingly more complex knowledge components with a longer time scale. A knowledge component (KC) relates task features, or conditions of application to a response (see examples in Table 2).

A simple KC example is:

> To state the meaning of the Chinese radical 日 in English, produce the word “sun.”

In this KC, the Chinese radical 日 and the goal of generating an English word are task features and the response is the production the English word “sun.”

Because knowledge components are unobservable, they are hypotheses about the nature of students’ capabilities that must be discovered from behavior on tasks or “assessment events”. In other words, KCs are not pre-determined by instructional designers, but can be empirically derived from sets of tasks that instructional designers can specify (either to characterize instructional objectives or pre-requisites of a course). Some simpler KCs can be directly tied to a single task, for instance, the KC above corresponds with the task “What is the meaning of the Chinese radical 日?” for which the answer is “sun”. For such simpler KCs, the task features are external in the world (as mediated by perception) and the responses act on and change the external features, as in a motor response.

However, in more complex or “integrated” KCs, the features can be internal, in the mind (e.g., inferred features of a new goal), and the responses can be internal mental actions. An example of such an integrated KC is:

> To find the area of irregular shape made up of regular shapes <S1> and <S2>, first find the areas of <S1> and <S2> and then add the results.

While knowledge may well be non-symbolic in its brain-based implementation, for purposes of scientific analysis, we describe knowledge components in a symbolic format, but distinguish whether the KC represents an implicit association, skill, or procedure that a student cannot verbalize and an explicit concept or procedure that a student can verbalize (cf., Alibali & Koedinger, 1999; Dienes & Perner, 1999). We employ a condition-response format to emphasize that knowing is not just about knowing what or knowing how, but knowing when. Indeed, learning the conditions of application of knowledge, the "when", may be more difficult than learning possible responses, the "what" (cf., Gobet & Wood, 1999; Zhu, Lee, Simon and Zhu, 1996).

Table 3 illustrates a classification of knowledge components using the following dimensions:

- The TASK FEATURES or conditions of application can be either constant or variable.
- The RESPONSE can be either constant or variable.
- The RELATIONSHIP between the tasks feature(s) and response can be implicit or explicit.
- A RATIONALE justifying the relationship between the task features and response may or may not be known.

Table 3. Some Basic Knowledge Component Categories

Task Features	Response	Relationship	Rationale	Labels
constant	constant	implicit	no	association
constant	constant	explicit	no	fact
variable	constant	implicit	no	category
variable	constant	explicit	no	concept
variable	variable	implicit	no	production, schema, skill
variable	variable	explicit	no	rule, plan
variable	variable	explicit	yes	principle, rule, model

In the example KC above, the task feature (the Chinese radical 日) is a constant and the response (sun) is a constant. The relationship is explicit (i.e., students express it words), but there is no underlying justification for the relationship, it simply the result of conventions of the languages. As indicated in the last column of Table 3 (second row), such knowledge components are commonly called "facts." These labels are rough mappings to commonly used terms and not necessarily one-to-one with the cells in this taxonomy (e.g., some "rules" have a rationale, like theorems in mathematics, but other "rules" may reflect regularities of arbitrary conventions, like grammar rules). It is an interesting question whether or not all 16 cells in the 4-dimensional space exist; certainly we do not have unambiguous labels for all cells. Our point, however, is to use these dimensions to better understand sources of complexity in knowledge acquisition and potential dependencies or constraints on relevant learning processes and instructional methods.

Constant->Constant KCs. The relatively simple "fact" KCs are pervasive in learning, including, for instance, vocabulary facts in 2nd language learning[1]. The relationship is explicit, but there is usually is no rationale or justification (cognates are an exception). An example of such fact knowledge is that the Chinese character for "lao3shi1" translates to the English word "teacher". Acquiring such KCs is an important instructional objective in second language learning. Similar KCs are essential in middle school, high school and post-secondary mathematics and science, particularly, definitions of terms. For instance, "pi" is the ratio of a circle's circumference to its diameter.[2] These tend to be relatively minor components in post-elementary mathematics, but they can be significant barriers to learning. For example, student performance in the Assistment system (Feng, Heffernan, & Koedinger, 2009) reveals that errors in simple fraction multiplication word problems (e.g., "What is 3/4 of 1 1/2?") are often not about the math, but about students not knowing that "of" indicates "multiplication". Because such constant->constant KCs may be implicit associations for experts (i.e., they perform the translation without verbal awareness), they can lead to "expert blind spot" (Koedinger & Nathan, 2004). An expert can misjudge what makes learning difficult for novices because he is unaware of the knowledge he has acquired that makes some tasks easy for him (like comprehending an algebra equation), but not for students (cf., Biederman & Shiffrar, 1987).

Variable->Constant KCs. A second broad category consists of KCs with variable conditions of application and a single or constant response. KCs with this structure are essentially category-recognition rules. Some examples of such many-to-one mappings include article selection in English noun phrases, for instance, "to construct a noun phrase with a unique referent, use the article 'the'" (e.g., "The moon …"). Such categorical or classification knowledge exists in mathematics as well, for instance, any expression that indicates the quotient of two quantities is a fraction[3]. Again such KCs may be represented only implicitly in a student's (or expert's) mind in that the student may be able to accurately recognize what expressions are fractions, but cannot articulate how she does so. For most first language English speakers, article selection knowledge is implicit – they do it effectively and fluently, but cannot explain their choices. We say a KC is known explicitly when students can explain the concept as well as give the right response under the right circumstances.

Variable->Variable KCs. Perhaps the largest category of KCs has variable conditions of application and has responses that vary depending on the conditions. An example in second

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[1] Vocabulary KCs for words with explicit morphological markers (e.g., past tense of regular verbs in English, like "jumped") are not members of this constant->constant fact category, but of the variable->variable rule category (e.g., To form the past tense of <verb>, produce <verb> followed by "ed").

[2] Note that the explicit constant->constant relationship between "the ratio of circle's circumference to its diameters" and "pi" is arbitrary, with no rationale, but the explicit constant->constant relationship between "the ratio of circle's circumference to its diameters" and 3.14 does have a rationale.

[3] Note that knowledge of recognizing expressions as fractions (e.g., saying that "3/4" is a fraction but that "3-4" is not) is not the same as being able to state the definition of a fraction, which constant->constant KC (mapping "fraction" to "quotient of two quantities").

language learning is a rule for generating an English plural: To form the plural of a singular noun <N> ending in an “s” sound or a “z” sound, form the word <N> “es”. In mathematics and science, KCs which apply formulas to solve problems have a variable->variable structure, like: To find the area of a triangle with height <H> and base <B>, multiply <H> * <B> * 1/2. As with the other categories these KCs can be known implicitly, explicitly, or both. They can also vary in whether or not they have a rationale. The rules for creating plurals essentially arbitrary in that there is nothing about the world that forces them to be as they are and thus, different languages have different ways to create plurals. Some KCs, on the other, have rationales and are essentially determined by nature. For instance, the formula for the area of a triangle is a provable regularity of Euclidean spaces that are approximated in the real world. We hasten to note that just because some KCs have rationales does not mean that students know the rationales when they know the KC. Quite to the contrary, it is often the case that a student may know a KC (an implicit or explicit one) and not know its rationale. The importance of the rationale feature of kinds of KCs, however, is relevant in considering whether certain forms of instruction, like collaborative argumentation or accountable talk (see below) will have purchase for a particular kind of knowledge component. The framework suggests a hypothesis that for KCs without a rationale, instructional approaches like collaborative argumentation or accountable talk may not be productive.

The distinctions between implicit and explicit KCs and those with and without a rationale are important for a number of reasons, but one of them is that different forms of the same idea can reinforce each other (e.g., Ainsworth & Van Labeke, 2004; Blum & Mitchell, 1998; Mayer, 2001; Mayer & Anderson, 1991; Paivio, 1990; Rittle-Johnson, Siegler, & Alibali, 2001; Wiley & Voss, 1999). Knowing a KC explicitly as well as implicitly may mean a student is more likely to recall it. Further, KCs that have rationales support sense making strategies whereby the rationale, if known, can serve to reconstruct a partially forgotten knowledge component or better adapt it to unfamiliar (transfer) situations.

Another reason to make these distinctions is that they suggest different levels of complexities of KCs that may be associated with how difficult they are to learn. We discuss other kinds of KCs and sources of complexity further below, but first we present data from PSLC LearnLab courses that illustrate how the different kinds of KCs may yield different performance times.

KC Complexity, Time to Apply, and Time to Learn. KCs can vary substantially in complexity, both between and within the kinds of KCs discussed so far. Variations within a KC category can result from differences in the complexity of the conditions (i.e., how much encoding or recoding of the perceptual stimulus must be done) or the complexity of the response and how it is constructed (i.e., the distance from motor operations). Across KC categories, KCs with variable conditions or responses tend to be more complex than constant conditions or responses, knowing a KC both explicitly and implicitly is more complex than knowing it just implicitly, and similarly knowing the rationale of a KC as well as the KC itself is more complex than knowing a KC without a rationale.

We suggest two ways to ground and estimate KC complexity. The first is theoretical. Simply put, the more complex the description of the KC, the more complex is the KC. Making this idea precise is not easy, but note related efforts in AI to define “description length” (Rissanen, 1978).

It requires making difficult decisions about what is the primitive or "atomic" level of KC description. Like molecules that can be made up of other molecules, descriptions of KCs may reference other KCs. Structure mapping theory (Gentner, 1983) provides one reasonably precise version of this idea in which knowledge structures are described as layers of relations between relations with the lowest layer being relations of surface features. Recall that our analysis is focused on students taking a specific course and thus we stop unpacking knowledge components into smaller components when the smaller components are ones that students can perform with a sufficiently high accuracy and fluency[4].

Example Knowledge Component (KC) types	Time to correctly apply (initial -> later)
Chinese vocabulary KCs: const->const, explicit, no rationale – If the Chinese pinyin is "lao3shi1", then the English word is "teacher"	6 -> 3 secs
English Article KCs: var->const, implicit, no rationale – If the referent of the target noun was previously mentioned, then use "the"	10 -> 6 secs
Geometry Area KCs: var->var, implicit & explicit, rationale – If you need to find the area of a triangle, and the base <B> and the height is <H>, then compute 1/2 * <B> * <H>	14 -> 10 secs

Figure 2. Examples of different kinds of Knowledge Components and associated learning curves from student performance in units from three different courses, Chinese, English as Second Language, and High School Geometry. The curves on the right show the change in the average time for a correct application of a knowledge component (the y-axis) after successive opportunities to learn it (the x-axis). Data points are averaged over all students and knowledge components.

[4] In a KC analysis for a particular course, a KC should be unpacked into smaller KCs when incoming student performance on tasks assessing those smaller KCs is sufficiently lower than a high threshold success rate (e.g., 95%), or in some cases, than a desired reaction time even when success rate is above threshold.

A second grounding of KC complexity is empirical and is in terms of the difficulty students have in applying the KC, one measure of which is the time it takes for students to correctly execute the KC. In general, simpler KCs can be executed more quickly, as suggested by the learning curves displayed in Figure 2. In these curves the y-axis is the average time for correct entries in computer-based tutoring systems for science, math, and second-language learning. The average is over each student on each target knowledge component in a particular unit of instruction. The x-axis shows the number of opportunities that students have had to practice and learn these knowledge components (feedback, hints, or examples of correct entries are provided as needed by individual students). In general, students learn with more opportunities and this relation is reflected in Figure 2 where the curves go down, indicating faster correct performance as opportunities increase.

Figure 2 shows learning curves from a Chinese vocabulary unit, an English grammar unit on use of articles, and a Geometry unit on area of figures. The Chinese Vocabulary unit involves constant->constant KCs like the Chinese character for “lao3shi1” translates to the English word “teacher”. Students’ correct performance of these KCs, which involves retrieving the correct response and typing it in, takes about three to six seconds on average (see Figure 1a). The English Article unit involves variable->constant KCs, like “if the referent of the target noun was previously mentioned, then use ‘the.’” Students’ correct performance of these KCs, which involves retrieving the correct response and selecting it from a menu, takes about six to ten seconds (see Figure 1b). The Geometry Area unit involves variable->variable KCs like if you need to find the area of a circle with radius <R>, then compute 3.14 * <R> ^ 2. These typically involve retrieving or constructing the needed mathematical operations or formula and typing in the arithmetic steps to be taken (e.g., 3.14*3^2) or the final result (e.g., 28.26). Correct entry takes about ten to fourteen seconds (see the third row in Figure 2).

Other empirical methods for estimating KC complexity are possible. In addition to time for correct performance, student difficulty with KCs can also be measured by error rates, for instance, the error rate observed on an associated paper test item (c.f., Koedinger, Alibali, & Nathan, 2008) or, within our computer tutors, the correctness of a student’s first attempt on a step (associated with a KC) without asking for a hint. Tutoring systems can further provide automated dynamic assessments of student learning process and progress. An “assistance score” is the amount of assistance a student needs to eventually get a step correct and is computed in the DataShop by adding the number of incorrect entries and number of hint requests a student makes on the step (e.g., 2 incorrect entries and 1 hint yields an assistance score of 3). The time to complete a step, including all incorrect entries and hint interactions, can also be informative and is provided by DataShop.

Other Kinds of Knowledge Components and Complexity Factors. Table 3 indicates some, but not all, important features of knowledge components and their complexity. Other kinds of knowledge have significance in understanding academic learning processes and have been the subject of investigations within the PSLC. These include integrative knowledge (e.g., Case & Okamoto, 1996; Slotta & Chi, 2006), prerequisite conceptual and perceptual knowledge (e.g., Booth & Koedinger, 2008), probabilistic knowledge (e.g., Frishkoff, Pavlik, Levin, & de Jong, 2008), incorrect or shallow knowledge (e.g., Aleven & Koedinger, 2002; Chi, Feltovich, & Glaser, 1981; Clement, 1987; Minstrell, 2001) and others.

We do not have space here to discuss all of these, but it is worth emphasizing that many components of knowledge are not directly tied to a single behavioral pattern (e.g., like recalling an appropriate formula in a given situation), but can nevertheless be inferred from behavioral patterns across a variety of task situations. We call such a component an "*integrative knowledge component*" because it must be integrated (or connected) with other KCs to produce behavior (it is not "atomic" in the sense discussed above). Such components might also be referred to as "hidden" or "cognitive" because they are not easily inferred from direct behavioral data. A typical strategy for such inference is to use a subtraction methodology where performance differences in two related tasks implicate an integrative KC. For example, Heffernan and Koedinger (1997) found students were significantly worse at translating two-step algebra story problems into expressions (like 800-40x) than they were at translating two closely matched one-step problems (with answers 800-y and 40x). They hypothesized that many students are missing an integrative KC that is necessary to solve the two-step problems but not for the one-step, namely an algebra grammar rule indicating that more complex expressions (like 800-40x) can be made up of simpler ones (40x). Recent research supports this hypothesis by showing how instruction specifically designed to address learning of this hidden KC significantly improved student performance (Koedinger & McLaughlin, in press).

Integrative KCs have a condition or an action that is an internal mental state like deep feature condition not directly observable or completely accurately inferred by target students from the external problem scenario or activity context. The above algebra grammar rule is integrative in that it requires integration with other knowledge, other simpler grammar rules, to produce an external behavior, an output.

In section 5, we discuss how different kinds of knowledge may have bearing on whether or not an instructional principle is relevant and effective. For instance, it may be that when the target of instruction is probabilistic knowledge or rules with many exceptions, more explicit forms of instruction (e.g., instructional explanations or prompts for self-explanations) will be relatively less effective than they are for other kinds of knowledge. Teaching or eliciting explicit rules may be counter-productive because there are too many exceptions and the probabilistic knowledge patterns are better acquired simply through examples and repeated practice with feedback. The opposite may be true for integrative knowledge where more implicit forms of instruction (e.g., example study, repeated practice) may not be sufficient for the brain to induce the necessary deep features and mental conceptual structures that implement the integration. In this case, providing and eliciting explanations may be critical to break down the complexity of the learning task and externalize the more complex chains of credit and blame assignment needed to construct integrative knowledge.

The sources of input for learning, whether examples or verbal descriptions, have alternative interpretations and this ambiguity means that students will often acquire knowledge that is incorrect. Usually that knowledge has some validity in that it produces correct behavior in some circumstances. The difficulty students experience in acquiring target knowledge depends not only on the complexity of the knowledge itself, for which the taxonomy above well addresses, but is also affected by the *field* of alternative interpretations. These field effects may be competitive, making learning more difficult, or cooperative, and make learning easier. For instance, in learning the English article system, it is tempting to acquire the rule that "an" is used rather than "a" when the first *letter* of the following noun is a vowel. This rule is consistent with most example uses, but in fact is incorrect, "an" used when the first *sound* is a vowel (e.g., "*an* honorarium"). Learners are often drawn (perhaps much more often than we tend to recognize) to make reasonable (implicit or explicit) interpretations of one or more examples that are incorrect. In geometry, for instance, students appear to implicitly acquire a rule that angles that look equal in a diagram are equal (Aleven & Koedinger, 2002). This issue of field effects is clearly relevant to instructional design as some design choices are more likely to tempt incorrect interpretations (cf., Chang, Koedinger, & Lovett, 2004), some may reduce that likelihood (e.g., Chang, 2006; Paas & Van Merrienboer, 1994; Schmidt & Bjork, 1992; Shea, Kohl, & Indermill, 1990). Other instructional design choices may leverage cooperative field effects, like concrete representations supporting abstract knowledge acquisition (cf., Goldstone & Son, 2005; Koedinger & Anderson, 1998; but see also Kaminski, Sloutsky, & Heckler, 2008) or sources of inductive bias from prior learning (e.g., object bias in word learning) or other related knowledge. That an idea or skill may be mentally represented in multiple forms (e.g., example, implicit rule, explicit rule) and connect in multiple ways to related knowledge components (e.g., links to other modalities or to a rationale) is likely to be critical to robust knowledge acquisition and retention. Support for this claim comes from both human learning (e.g., Ainsworth & Van Labeke, 2004; Mayer, 2001; Mayer & Anderson, 1991; Paivio, 1990; Rittle-Johnson, Siegler, & Alibali, 2001; Wiley & Voss, 1999) and machine learning (e.g., Blum & Mitchell, 1998; Caruana, 1997; Muslea, Minton, & Knoblock, 2002; von Neumann, 1956) research.

3.2 Knowledge Component Complexity Guides Instructional Decisions

A fundamental KLI assumption is that many learning processes and instructional design decisions are not intrinsically domain-based, but are determined by the type of KCs being learned (cf., Sweller & Chandler, 1994; Wulf & Shea, 2002). While some types of KCs (e.g., constant-constant KCs) may be particularly prevalent in some domains (e.g., second language vocabulary learning), instructional principles should refer to KCs rather than to domains. For example, a hypothetical principle "drill and practice is not effective for mathematics" is at the wrong level of analysis because it does not describe what it is about mathematics that makes drill and practice unsuitable. Instead, we believe it is more productive to associate instructional design decisions with knowledge component types and associated learning processes, such as:

- Study-practice trials (e.g., Pavlik, 2007) support constant->constant fact encoding and memory strengthening
- Worked examples, tutored practice (e.g., Koedinger & Aleven, 2007), and apprenticeship-style model-scaffold-fade strategies (e.g., Collins, Brown, Newman, 1989) support inductive learning and compilation of implicit knowledge of variable condition categories and productions,
- Prompted self-explanations involve verbally-mediated reasoning chains that lead to construction of explicit knowledge of concepts and rules and to integrative knowledge that connects implicit and explicit knowledge components

- Accountable talk moves (Michaels, O'Connor, & Resnick, 2008) by teachers, like "Is this always true?", prompt verbally and socially-mediated reasoning chains that can lead to more elaborated integrative knowledge connecting rationales to explicit concepts and principles

It remains an open question whether domain specificity constrains the form of knowledge components in a deep qualitative way or only in a quantitative way. To the extent that domains reflect biological constraints, one might consider the possibility of qualitative differences. Thus first language learning would have constraints that make it easier (KCs are partly given) than say algebra learning. However, second language learning may cede this advantage, becoming more like algebra learning than first language learning. These are important questions that require their own research, but for our purposes we believe domain-specific knowledge components are necessary as a starting point and the connection across domains is to be determined.

In the following sections we discuss the wide variety of kinds of (unobservable) learning processes that mediate between kinds of (observable) instructional events and the resulting acquisition of different kinds of (unobservable) knowledge (see Figure 1).

4. Learning: Toward a Taxonomy of Processes for Knowledge Acquisition and Improvement

There are many robust learning processes and a goal for the macro-level of the KLI framework is to formulate a useful taxonomy for them. Our theoretical framework recognizes three major categories of learning processes, listed from simpler to more complex:

A. ***Fluency building processes***: Implicit learning processes involved in memory strengthening and streamlining of knowledge to produce more automatic and composed ("chunked") knowledge. Fluency building can be conceived as making the link between the condition and response of a KC more direct, more consistent, faster to response, and more resistant to interference.
B. ***Refinement processes***: Non-verbal learning processes that improve the accuracy of knowledge including generalization, discrimination, classification, categorization, causal induction, non-verbal explanation-based learning (short non-verbal deductive chains). Refinement can be conceived as modifying the condition (or response in the case of a variable response) part of a KC to make it more accurate, that is, more likely to produce desirable behavior under appropriate circumstances.
C. ***Sense making processes***: Explicit (language-mediated) learning strategies wherein students deliberately try to understand the instruction or engage in higher-level thinking including comprehension of verbal descriptions of KCs, verbal explanation-based learning, scientific discovery, explicit rule-mediated deduction. Sense making can be conceived as making links between implicit and explicit forms of knowledge or between a KC and its rationale.

Figure 3 illustrates how these different learning processes can lead to different kinds of knowledge changes and ultimately to measurable robust learning outcomes.

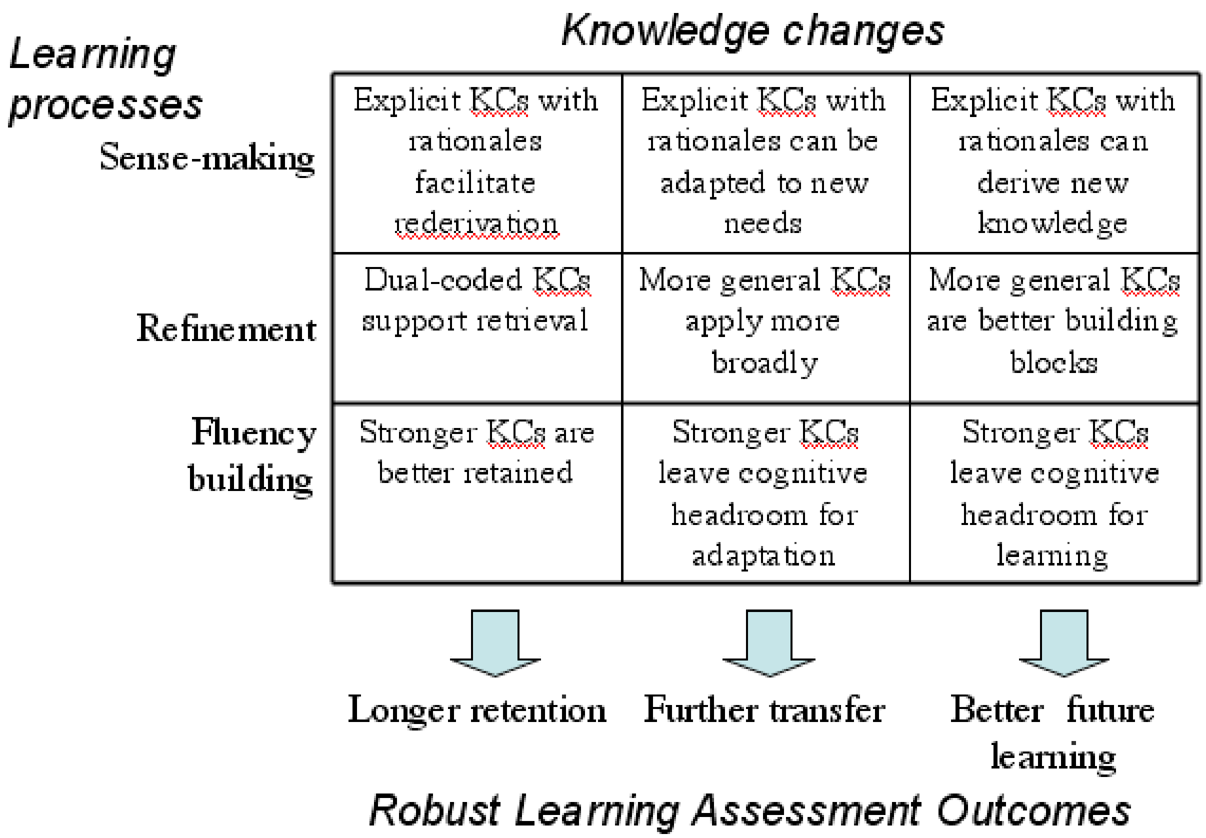

Figure 3. How different classes of learning processes change knowledge to yield different robust learning outcomes.

Fluency building processes. The brain is constantly engaged in creating and strengthening of connections between the conditions and responses of knowledge components in use. Strengthening operates all throughout learning from the initial formation of a KC to each time it is subsequently used. It operates on all kinds of KCs from the simplest implicit constant->constant associations to the most complex variable->variable schemas or model structures. Even after a knowledge component becomes well established (reliably applied by the student) and its information content is not changing much, fluency-building processes continue to act, leading to fast and effortless performance. There are two principle processes underlying fluency gains, knowledge compilation and strengthening, although it is not clear they are different as opposed to reflecting different levels of analyses. Knowledge compilation leads to faster *application* of a knowledge component. In compilation, an initial declarative encoding of knowledge components is proceduralized into a directly executable form (Anderson & Lebiere, 1998) and chains of small knowledge components may be composed into a single larger KC. In strengthening, knowledge components become more available in memory with repeated use, resulting in faster and more reliable *retrieval* of KCs.

Part of the study of fluency is determining how it accelerates further learning (see the lower right cell in Figure 3). Although it is widely believed that mastery of the parts facilitates more efficient sense making of the whole, it is not clear exactly how this acceleration works. Such open questions that address knowledge-based influences on learning are at a level of analysis that is critical for engineering effective academic courses, but which falls between lower analyses cognitive psychology and cognitive neuroscience and high level analyses of education research -- in other orders, at a level of analysis the KLI framework is intended to address.

Refinement processes. Refinement processes involve modifications to knowledge components, particularly to the conditions or features under which the knowledge is retrieved and should be applied. Like fluency building processes, knowledge refinement processes can occur throughout the "lifetime" of a KC (or of a cluster of related KCs). Refinement processes are relevant to Variable->Constant and Variable->Variable KCs. These processes modify the condition part of the KC and may add a missing relevant feature to the condition of a knowledge component (a "discrimination") or remove an irrelevant feature (a "generalization"). For instance, consider a typical novice knowledge component in geometry: "If angles look equal, then they are equal" (Aleven & Koedinger, 2002). While this knowledge component can yield correct answers, it is incorrect in general. A student may *refine* this knowledge component by removing the irrelevant feature "angles look equal" or by adding a relevant feature, like "angles opposite each other in crossing lines" or "angles that are base angles of an isosceles triangle". Such refinement leads to a knowledge component with higher "feature validity", meaning it is now more likely to be applied when appropriate and not applied when inappropriate. Early learners may make errors in the refinement process by adding an irrelevant feature condition to a knowledge component (e.g., because in an example the angles that are concluded to be equal, do look equal, a student may incorrectly induce that looking equal is a relevant feature) or by failing to add a relevant feature (e.g., not notice that the base angles in an isosceles triangle are the ones across from the equal sides).

While fluency-building processes involve core mechanisms of the cognitive architecture, such as memory strengthening, refinement processes make use of existing knowledge, such as more elemental KCs that may be composed into the condition or response of a larger KC. They also draw on cognitive resources and take time to execute. Unlike the sense-making processes discussed next, refinement processes can operate without explicit awareness and cannot be directly verbalized. Specific kinds of refinement learning mechanisms (non-verbal learning processes that improve the accuracy of knowledge) can be found in the cognitive science and machine learning literatures including generalization (e.g., Catrambone, 1996; Hummel & Holyoak, 2003; Shepard, 1987), discrimination (e.g., Chang, 2006; McClelland, Fiez, & McCandliss, 2002; Richman, Staszewski, & Simon, 1995), perceptual chunking (e.g., Gobet, 2005; Servan-Schreiber & Anderson, 1990), classification and categorization (e.g., Blum & Mitchell, 1998; Collins & Singer, 1999; DiBello, Stout, & Roussos, 1995; Medin & Schaffer, 1978; Quilici & Mayer, 1996), non-verbal explanation-based learning (Mitchell, Keller, & Kedar-Cabelli, 1986).

We use refinement to refer to feature changes whether they are changes to explicit (declarative or verbal) knowledge components, or changes to implicit (procedural or non-verbal) knowledge components. Learning from examples or by experience may result in feature refinements that students cannot verbalize. For instance, first language learners acquire the features for correct choice of articles, like "a" and "the", without being able to articulate the explicit rules for article choice. Even second language learners, as well as math and science learners, engage in such non-verbal feature refinement and produce implicit knowledge. Non-verbal feature refinement can also produce explicit, verbal knowledge. Such cases are indicated when a student can express a rule with correct features (e.g., "it is the *base* angles of an *isosceles* triangle that are equal"), but cannot explain how they learned (came to know) those correct features. When a student can explain, they are engaging in sense making, which is discussed in the next section.

Sense-making processes. Sense-making processes are robust learning strategies wherein students try to understand the instruction or engage in higher-level, language-mediated thinking to create knowledge, perhaps independently of instruction. Sense-making processes involve explicit reasoning and include comprehension strategies, self-explanation, and social argumentation. While sense making can focus on task features and responses, it plays a key role in transforming implicit relationships into explicit relationships, thereby transforming constant-constant associations into facts, variable-constant categories into concepts, and variable-variable productions into rules. Similarly, when sense-making focuses on the rationale for a feature-response relationship, it transforms variable-variable rules into principles.

As with fluency building and refinement, sense making may occur at any point during the lifetime of a KC. However, unlike fluency building processes, which are always active (i.e., running as a background process in the brain's hardware architecture), sense-making strategies are deliberate and occur only when a student chooses to engage in them. Unlike refinement learning mechanisms, which are non-verbal, sense-making strategies are explicit, in the sense of being supported by the use of words or external symbols (whether sub-vocalized, written, or spoken in social dialogue). Sense-making processes include explicit comprehension strategies (e.g., Graesser, McNamara, & VanLehn, 2005; Palinscar & Brown, 1984), verbally-mediated self-explanation (e.g., Ainsworth, S., & Loizou, 2003; Chi, Bassok, Lewis, Reimann & Glaser, 1989; Lewis, 1988; VanLehn, Jones, & Chi, 1992), explicit hypothesizing and scientific discovery processes (Levine, 1966; Klahr & Dunbar, 1988), deductive proof (e.g., Stylianides & Ball, 2008; Stylianides & Stylianides, 2009), and explicit argumentation like collaborative discourse (e.g., Asterhan & Schwarz, 2007; 2009).

Summary. Characterizing the range of learning processes is important because they provide the causal link between instructional methods and changes in student knowledge. The KLI framework suggests there are likely to be important dependencies between kinds of knowledge, learning processes, and choices of most effective instructional methods. We have indicated above some dependencies between knowledge and learning processes, for instance, that fluency building processes may be most important for learning simple Constant->Constant components without a rationale whereas sense making processes may be most important for learning more complex Variable->Variable components that have a rationale (and may be irrelevant for components without a rationale). Fluency-building processes may also be relevant for more complex Variable condition components that, for instance, may not last in long-term memory without appropriate repetition. Similarly, refinement processes are also likely to be relevant for the kinds of complex integrated and interconnected KCs (with rationales) produced by sense-making processes. These observations suggest a potential asymmetry whereby simpler learning processes (fluency and refinement) may support complex knowledge but complex learning processes (e.g., argumentation) may not be relevant to simple knowledge (e.g., arbitrary Constant->Constant associations).

In the next section we describe examples of some instructional principles that characterize methods that have been experimentally demonstrated to yield better robust learning efficiency outcomes then reasonable alternatives. After that, in section 6, we provide a more detailed analysis of one of these principles. Finally, in section 7, we return to the issue of dependency and, in particular, hypothesize a general relationship between knowledge component types and instructional methods.

5. Instructional Principles and Hypotheses About their Effectiveness

The main question for this section is what kinds of instructional events yield robust learning in an efficient way? We are not only concerned about robust learning outcomes, but the instructional time in which they are achieved. We seek principles that achieve greater robust learning outcomes without taking more time or that achieve equivalent robust learning, but in significantly less time. Table 4 shows a list of instructional principles from simplest to most complex. The simplest, Optimized Scheduling, has tended to be used with the simplest kinds of knowledge components, constant->constant facts (e.g., Pavlik, 2007). The most complex, Accountable Talk, has tended to be used with the most complex kinds of knowledge components, explicit principles with rationales. We next describe some of these in more detail.

Table 4. Some examples of instructional principles, roughly from simpler to more complex.

Instructional Principle	Description	Example References
Optimized Scheduling	Selection of practice instances based on prior statistics and on each students' experience with each target KC.	Pavlik (2007)
Timely Feedback	Providing an evaluative response (e.g., correct or incorrect) soon after a students' attempt at task or step	Corbett & Anderson (2001)
Feature Focusing	Instruction leads to more robust learning when it guides the learner's attention ("focuses") to valid or relevant features of target KCs	Dunlap et al (under review)
Worked Examples	Students learn more efficiently and more robustly when more frequent study of worked examples is interleaved with problem solving practice as opposed to practice that is all problem solving.	Sweller & Cooper (1985)
Prompted Self-Explanation	Encouraging students to explain to themselves parts of instruction (steps in worked example or sentences in a text) yields more robust learning than not prompting or providing such explanations to students.	Chi et al (1994); Haussmann & VanLehn (2007)
Accountable Talk	Encouraging classroom talk that is accountable to accurate knowledge, rigorous reasoning, the classroom community by using some six talk moves (question and response patterns) leads to more robust learning.	Michaels, O'Connor, & Resnick (2008)

Optimized Scheduling

This principle involves applying an instructional schedule that has been ordered to maximize robust learning. Optimized scheduling involves maximizing instructional efficiency (i.e., robust learning gains per instructional time spent) by mathematically deriving when a student should repeat practice of a knowledge component. The time interval between practice opportunities of a KC is optimal (neither too short or too long) when it best balances the benefit of enhanced memory strength due to retrieval at a long interval (spaced practice) and the cost of time to retrain due to retrieval failure, which is more likely at a long interval. Mathematical models may be used to produce optimized schedules by computing the knowledge component that will be most efficiently learned if practiced next.

The spacing recommendation in the “Organizing Instruction” practice guide (Pashler et al, 2007) describes a simpler version of this principle, namely that spaced practice leads to better long-term retention than massed practice. The more specific principle above builds on Pavlik’s (2007) observation that much past research has not controlled for time on task and has thus have underestimated the benefit of shorter practice intervals early in KC acquisition.

In addition to the optimized scheduling studies of Pavlik (2007), other instances of this principle include studies of the knowledge-tracing algorithm used in Cognitive Tutors (Corbett & Anderson, 1995; Cen, Koedinger, & Junker, 2007) and adaptive fading of worked examples into problems to solve (Salden et al, 2008).

Feature Focusing

This principle involves encouraging students to focus on cues features that are valid for the targeted knowledge components. By directing students attention to key features, feature focusing instruction may help students to more quickly learn those knowledge components most important for the goals of learning. More generally, attention focusing may also result in students spending more time during a learning event on a particular knowledge component and thus increase its strength.

An example of feature focusing comes from learning to read Chinese, whose characters often are compounds, consisting of two components. Often these components (radicals) provide cues to pronunciation and meaning. For example, consider the compound character 晴, which is translated as fair weather. On the left is a semantic radical 日 that means “sun” and on the right is a phonetic radical 青 that is pronounced “qing”. Knowing that 日 means ”sun” is useful in learning the meaning of this compound and others that contain it. Feature focus directs attention to the form for “sun” in association with is meaning. Standard Chinese reading instruction tends not to do this, emphasizing instead the meaning of the character as a whole. However, the research indicates that focusing on the feature of component form-meaning association supports learning of characters (Taft & Chung, 1999). In a particular implementation of this principle, Dunlap and Perfetti (under review) found that highlighting the semantic component as a student moves the computer mouse over it improves learning the character.

Worked Example Principle

In a worked example, students are given a problem description along with a step-by-step solution to the problem and our asked to study or self-explain the solution. Since Sweller and Cooper (1985), countless studies have shown that interleaving worked examples and problems to solve leads comparable procedural learning and deeper conceptual learning than straight problem solving, and is more efficient, requiring less average learning time. Since this instructional principle relies on explicit reasoning, it is applicable to KCs with explicit rationales, and to sense-making and refinement learning. However, for more advanced students, interleaved worked examples are less efficient than straight problem solving (Kalyuga, Chandler, Tuovinen and Sweller (2001), demonstrating that this may not be an effective instructional principle for increased fluency. Section 6 below provides an expanded analysis of this principle.

Prompted Self-Explanation Principle

When students are given a worked example or text to study, prompting them to self-explain each step of the worked example or each line of the text causes higher learning gains than having them study the material without such prompting (e.g., Aleven & Koedinger, 2002; Chi, de Leeuw, Chiu, & LaVancher, 1994; Renkl, Stark, Gruber, & Mandl, 1998). Hausmann and VanLehn (2007) is a good example of the power of prompted self-explanation. They found that prompting students to self-explain while solving a physics problem produced more learning than providing them with high quality explanations to study. When it comes to explaining physics examples, it appears it is better to do it yourself, even if you get it wrong, than to study someone else's explanations.

This principle provides a good example of how the effectiveness (or even relevance) of a principle may be dependent on the nature of the target knowledge components. When both implicit and explicit versions of knowledge are objectives of instruction, prompting self-explanation can pay off doubly by both 1) strengthening explicit forms of knowledge and 2) providing redundant support (co-training) for acquiring implicit forms of knowledge. The second reason relies on a learning process assumption that self-explanation is explication process and functions in addition to implicit learning processes. That explicit knowledge is an instructional objective tends to be true for much of math and science (i.e., students are expected to able to state principles, like Newton's laws, and provide explanations of solutions) and not to be true in language acquisition (e.g., it is not critical that a fluent speaker is able to explain why "a" is used before "fluent speaker").

Prompting students to self-explain may not aid learning for some kinds of KCs. Such a possibility would be consistent with the verbal overshadowing effect (Schooler et al, 1997), in which performance on tasks that require rich perceptual discrimination (e.g., face recognition) diminishes when subjects describe the perceptual experience. We might consider that something similar will occur when knowledge components are relative simple in the ways we have described here; for example, a knowledge component that is implicit and lacks a deep rationale, is probabilistic, and depends primarily on perceptual information. Hints in that direction come from a PSLC study of learning the English double dative construction (Friskoff, Levin, Pavlik, & de Jong, 2008). English allows speakers to say either "John gave the book to Mary" or "John gave Mary the book". However, rather than a set of rules, native speakers' choices about which noun to put first is subject to a web of factors that are implicitly weighted (Bresnan et al, 2007). These factors can be translated into instructional heuristics, and when Frishkoff et al (2008) did this, they found that English second language learners subsequently made choices more in line with those of native speakers. However, when they provided these heuristics as feedback following some degree of learning by correctness feedback only, they found it was not helpful. This might imply that verbalization—in this case not by the learner—can be helpful or harmful as a function of specifically how it connects to learning that is proceeding implicitly. Self-explanation also might function within a similar space, supportive of learning in general but sometimes interfering with other learning processes.

Summary. In the prior section, we illustrated how different kinds of learning processing may be relevant for different kinds of knowledge components. Here we have illustrated how different instructional methods evoke different learning processes. To the extent that such dependency relationships are indeed strong than we would expect that instructional principles will not apply universally but will be dependent on the kind of knowledge components that are the targets of instruction. We illustrate such dependency in the next section by reviewing in some detail how one instructional principle—the worked examples principle—can be understood in terms of the KLI framework.

6. Expanding the Worked Example Principle with a Knowledge Component Analysis

This section provides an example for how the KLI Framework can be employed to better understand an instructional principle by focusing on how it operates in learning events at the knowledge component level. Such a focus provides perspective on alternative theoretical accounts of why this principle works and can yield a more clear specification of the boundary conditions on the applicability of the principle. The principle has been characterized as applicable only to novice learners, but we suggest a more specific boundary condition based on a knowledge component analysis.

Problem solving has long been a canonical learning activity, but in a seminal paper, Sweller and Cooper (1985) compared straight problem solving to a condition in which students were asked to study worked-out solutions to every other problem. Students received no feedback on their explanations of the worked examples. In problem solving, students received whole answer accuracy feedback on their solutions, and were required to continue working until they generated a correct solution or until a time limit expired and they were shown a correct solution. The authors found that the interleaved worked example condition was always more efficient, requiring substantially less average learning time, and often more effective, with students in the worked example condition making fewer errors on a post test. Interleaved worked examples can also support better transfer to novel problems than straight problem solving (Cooper & Sweller, 1987; Ward & Sweller, 1990). The benefits of interleaved worked examples have been replicated repeatedly (Carroll, 1994; Paas, 1992; Paas and van Merrienboer, 1994; Quilici & Mayer, 1996; Tarmizi, R.A. & Sweller, J. 1988;Trafton and Reiser, 1993).

A second landmark paper revealed a critical expertise reversal effect. Kalyuga, Chandler, Tuovinen and Sweller (2001) found that for novices, interleaved solution analysis and solution generation led to better learning than solution generation alone, but for more experienced students, straight problem solving yielded better learning than interleaved worked examples and problem solving. This result has also been replicated (Kalyuga, Ayres, Chandler & Sweller, 2003; Kalyuga, Chandler & Sweller, 2001).

These two results yield an initial principle for worked examples at the grain size of whole problems: *interleave worked examples and traditional problem solving for novices, then switch to straight problem solving for more advanced students.* A goal of the KLI framework is to understand this principle and how to apply it at the level of knowledge components, rather than whole problems, to examine the definition of novices, and examine theoretical perspectives on the worked example effect.

6.1 Transition to a Knowledge Component Analysis of the Worked Example Principle

In the earliest worked example research the basic unit of activity was the complete solution: students were asked to study full worked examples and in problem solving were given accuracy feedback and correct answers based on full solutions. Before examining the worked example principle within the KLI framework, we briefly summarize some variations on worked example and problem solving procedures that have shifted the analytic focus to finer-grained solution components.

Incomplete Problems. When presented complete worked problem solutions, students may fail to carefully study each step (Chi et al., 1989; Renkl, 1997; Renkl, Stark, Gruber & Mandl, 1998). In an initial response, several studies present students with partial problem solutions instead of complete solutions (Paas, 1992; Van Merriënboer, Schuurman, de Croock, & Paas, 2002). In this paradigm, students are asked to study the partial solution and to complete the remaining steps, thereby ensuring that students attend to those steps. These studies generally find that working incomplete problems is more effective than conventional problem solving, but not more effective than self-explaining complete worked examples.

Fading. The problem-completion paradigm inspired a new method for making a smooth transition from full worked examples to full problem solving, called "fading." Renkl and colleagues compared interleaved worked examples and problem solving to a "fading" condition in which students first self-explain all steps in a worked example, then start to solve some of the steps and explain other given steps (examples), and finally solve all steps (Renkl, Atkinson & Maier, 2000; Renkl, Atkinson, Maier & Staley, 2002; Atkinson, Renkl & Merrill, 2003). They found that fading was more effective than interleaved worked examples, yielding better learning outcomes with the same learning time.

Knowledge-Component Support in Problem Solving. The standard control condition in previous worked-example research has been to ask students to solve a serious of problems equal in number to the combination of examples and problems in the treatment, for instance, six problems rather than 3 examples interleaved with 3 problems. After a student completes a problem (whether successfully or not) they are given an example of a correct solution, in other words, they receive whole-answer feedback. This control condition is relatively weak (cf., Schwonke, Renkl, Krieg, Wittwer, Aleven, & Salden, 2009) given that problem solving with the step-by-step assistance provided by Cognitive Tutors has also been shown to be more efficient and effective than problem solving with whole-answer feedback in mathematics and programming (Corbett & Anderson, 2001). Students receiving immediate step-by-step advice in learning finished in one-third the time required by students receiving whole-answer feedback and made 40% fewer errors on paper-and-pencil posttests. Several PSLC studies have compared worked examples with step-by-step support for problem solving in intelligent tutors. As a result, an important question is whether problem solving with step-by-step support becomes more effective and/or more efficient if it is integrated with worked examples. Several recent studies have examined this issue in the context of intelligent tutors for math and science.

Anthony (2008) conducted a classroom study in which the standard Algebra Cognitive Tutor was compared with a version that included annotated worked examples with problem-solving tasks as a means of providing students more information because detailed feedback was unavailable. The results revealed that students who learned with examples attained better long-term retention and also showed that both the examples Tutor condition and the regular Cognitive Tutor condition experienced a similar amount of mental effort.

McLaren et al (2008) replaced half the problems in an intelligent chemistry tutor with worked examples and found that, even with this stronger problem-solving comparison condition, the interleaved example condition was more efficient. Across three studies, one at the college level and two at the high school level, the authors found students learned just as much but in about 20% less time. A careful analysis of log data provides an explanation for both the similarity in final learning outcomes and the significant difference in learning times. Students in the Problems Condition can effectively turn problems into examples by using the tutor to *create* examples through requesting all of the hints the tutor provides, the last of which, the so-called "bottom-out hint", provides the answer (which is exactly the same information provided by the example). In the Problems Condition, students requested bottom-out hints much more often when working on the first of the isomorphic example-problem pairs. The extra time in the problems condition is also isolated to these first problems – students in the Examples Condition take much less time on the matched activity (which is an example rather than a problem).

Schwonke et al. (2009) compared problem solving with step-by-step support with faded worked examples in the Geometry Cognitive Tutor. Students in both the worked-example and problem-solving conditions explained each problem-solving step by entering a corresponding geometry principle. In the worked example condition, the problem-solving steps were systematically faded. Students completed faded steps with conventional Cognitive Tutor support - immediate feedback and advice as needed. These authors replicated the efficiency results of McLaren et al (about 20% less time need in the examples condition), but also found a robust-learning effect on a test of conceptual transfer.

6.2 Theoretical Interpretations.

At least three types of theories attempt to explain why worked examples are more effective and efficient for novices. All three theories agree that novices can develop a deeper understanding of problem situations in explaining worked examples than in problem solving, but vary in their explanatory mechanism. Two explanations hinge on general cognitive capacity and motivation. A third type of explanation focuses on individual knowledge components.

Cognitive Load Theory. The theoretical framework that drives much of the worked example research is Cognitive Load Theory (Sweller, 1994; Van Merriënboer & Sweller, 2005). Human processing capacity is limited, and this theory argues that much of the cognitive load in problem solving is extraneous to robust learning. In early problem solving, novices rely heavily on means-ends analysis and trial-and-error strategies. While these processes are useful for solving novel problems, they require cognitive resources that cannot then be used (or be used as much) to engage in refinement or sense-making processes like self-explanation. Thus, novices are less likely to develop a deep understanding of the domain while engaged in pure problem-solving practice.

Task Demands and Goal Orientation. A second theory focuses on implicit task demands of problem solving and worked examples. While the goal of problem solving is learning, the putative goal is on solving the problem rather than understanding and learning from the problem. Katz, Lesgold, Eggan and Greenberg (1996) have shown that when students reach an impasse in problem solving, they generally just want to know what to do next and are uninterested in branching into an extended explanation of the deep knowledge that guides the next step. It seems that exclusive problem solving practice may produce a goal orientation (e.g., Elliot, 2005) toward performance (get the problems learned) rather than mastery (learn the principles and skills).

Filling Knowledge Component Gaps. A third theory proposes that students are more likely to notice misconceptions or correctly fill gaps in their knowledge when they are self-explaining examples than when they are solving problems, because *more information is available* in examples than problems. Since a complete and correct solution is not available in problem solving, students can apply incorrect knowledge without realizing it (resulting in an incorrect solution). In addition, when students do recognize knowledge gaps, they are more likely to construct appropriate KCs in the worked example condition, because their reasoning is more constrained by the presence of the solution. Vanlehn, Jones and Chi (1992) embodied this theory in a formal computational model called *Cascade*, which successfully modeled verbal protocols of student self-explanations. Converging evidence for this model in related research in which Corbett and Anderson (2001) compared immediate feedback and whole-answer feedback in the Lisp Programming Cognitive Tutor. In the conventional Cognitive Tutor immediate feedback condition, students received step-by-step accuracy feedback and, upon request, received sufficient advice to complete the step. In the whole-answer condition, students received accuracy feedback on their completed solutions and upon problem completion, were shown a complete correct solution. This study found that students in the whole-answer condition took three times as long to complete the programming problem set, they failed to solve 30% of the problem solutions on their own and, while they received whole-answer feedback, were 25% less accurate on a paper-and-pencil posttest.

These three explanations may not be mutually exclusive. Indeed, it appears both the Cognitive Load and Knowledge Component Gap theories rely in part on the implicit task demands explanation. Since students in the problem-solving control condition are provided with whole-solution feedback, which is a worked example, students could choose to self-explain each step in this solution, just as in the worked-example condition. In that case, they would learn just as much (even if wasting some valuable instruction time on some of the problems they cannot solve). But, many studies show that learning outcomes are worse. Apparently, like the students in Katz et al (1996), students do not view their learning task as the same when presented with whole-solution feedback as they do when presented with an identical worked example (without seeing it first as a problem). A possible alternative amendment to Cognitive Load theory is that the problem-solving experience creates residual cognitive load (fatigue) that carries into that whole-solution feedback phase such that students do not have sufficient capacity to effectively study the solution feedback[5]. Similarly, it may be argued from the Knowledge Gap theory that the relative lack of information for learning during problem solving leaves the learner to make shallow

[5] Note that such a fatigue notion of load that carries over from one task activity to the next is distinctly different from published Cognitive Load theory, which states that storage of goals during means-ends analysis creates undesirable load *during* problem solving.

inferences and that students replicate these inferences during whole-solution feedback study rather than engaging in deeper self-explanation.

Comparing the Theories in Explaining Example Fading. Some research has begun to contrast these theories in the context of faded examples, that is, giving students answers to all steps initially (example steps) and then gradually "fading" the answers away so student must provide them (problem steps). Jones and Fleishman (2001) ran the Cascade model over a set of Newtonian physics problems, asking it both to explain worked examples and solve problems. These simulations replicated the benefit of faded examples over complete worked examples on subsequent problem solving. In addition, the authors found that Cascade learns more effectively when the problem-solving steps that are faded are the ones that introduce new KCs. In a subsequent study, the authors found that this Cascade model provides a good fit to verbal protocol files of students self-explaining faded examples (Fleischman & Jones, 2002).

While this Cascade modeling work supports the Knowledge Gap theory, a finding by Renkl et al (2002) seems more compatible with Cognitive Load theory. They found that *backward* fading (fading the last step, then the last two steps, etc.) is more efficient and effective than *forward* fading (fading the first step, then fading the first two, etc). This result supports Cognitive Load Theory under the assumption that student's cognitive load is lower at the end of a problem than at the beginning, so the student is better able to learn from faded steps at the end of a problem. However, this result is ambiguous because, as Renkl, Atkinson and Große (2004) pointed out, the knowledge component content of faded steps is typically confounded with the position of faded steps. So these researchers conducted two studies in which the position of a faded step was manipulated independently of the specific knowledge component required for the faded step. The results were consistent with the Knowledge Component theory rather than the Cognitive Load theory; students test performance was reliably predicted by the specific KC that governs a faded step, but not by the position of the faded step. We note that this result is also inconsistent with a pure task-demand explanation of the benefits of self-explanation of worked examples. Since the task demands appear identical in the various conditions, a task-demand theory would predict no differences among the conditions.

6.3 Defining Novices: A Knowledge Component Analysis.

The working definition of novice in most worked example research has been students who have just received instruction and are just beginning to apply it. It is not clear how well this definition works in practice given that students in classrooms can vary greatly in their prior knowledge. For instance, students in an 8^{th} grade math course may vary from 4^{th} grade to 10^{th} grade competence levels. Kalyuga and Sweller (2004, 2005) addressed this variability by developing pretests that efficiently categorize students as novices or advanced students and showed that channeling the novices into worked examples and the advanced students directly into problem solving yielded more efficient learning outcomes.

An alternative knowledge-component based solution, developed by Salden and colleagues (Salden, Aleven, Renkl & Schwonke, 2008), is to monitor students growing knowledge of each knowledge component and to fade from giving that KC as an example to requiring student application when the student has demonstrated some understanding of it. The difference between the two approaches highlights a difference in the definition of "novice". The usual notion, consistent with Kalyuga and Sweller, is that being a novice is a general state of a learner with respect to a *domain*. The KLI framework alternative is that a learner is a novice with respect to particular knowledge components, namely, those KCs for which the student has had little prior experience (or, more precisely, those KCs that correspond with tasks on which the student has demonstrated low performance). One approach for assessing and adapting to individual students' prior experience of a specific KC is to fade instruction on that KC when a student can reliably generate an explanation of an example application of that KC. Salden, Aleven, Renkl & Schwonke (2008) tested this approach in laboratory and *in vivo* experiment in the Geometry domain by comparing such individually adaptive fading with both fixed fading and pure problem-solving. They found evidence that individualized fading yields more robust learning than the other two conditions.

7. Linking knowledge analysis and instructional principles

Across the wide variety of instructional experiments, we find that simpler instructional event types (involving less time, less feedback, less verbalization and reasoning) tend to be associated with simpler knowledge components (involving less time, less complex conditions, response or integration with related KCs). Table 5 illustrates this relationship.

Table 5. Which instructional events or principles work effectively for which kinds of knowledge component goals? Is there a "complexity correlation"? The rows and columns in this table are organized from simple to complex. A "+" indicates the corresponding instructional principle has been experimentally demonstrated to enhance robust learning efficiency for the corresponding kind of knowledge components. A "0" means an experiment found no difference.

Instructional Events (simpler on bottom)	Chinese vocab	French articles	English articles	Algebra eq	Geometry rules	Chemistry rules	Help seeking skls	Physics prinpls	Chem models	Pressure concept
Accountable Talk										+
Collaboration				0					?	+
Self-explanation			0	+	+			+		?
Worked examples				+	+	+				?
Diagram coordination					+				+	
Feature Focusing	+	+								
Feedback		+		+			+			
Optimal Scheduling	+				+					

Knowledge Components (simpler on left)

Is this association for good reason? Do complex forms of instruction not work well for simple knowledge goals? Do simple forms of instruction not work well for complex knowledge goals?

We can contrast the "match" hypothesis and the "embedded complexity" hypothesis. The match hypothesis suggests that an effective instructional method will be about as complex as the target knowledge components, not much more or much less. This hypothesis is consistent with the apparent association in Table 5. The embedded complexity hypothesis suggests that an effective instructional method can be simpler or as complex, but not more complex than the target knowledge components. This hypothesis implies that applications of instruction types to knowledge types in the lower-right of Table 5 are effective, but that those in the upper-left are not.

What is the theoretical support for these alternative hypotheses? We need to consider the learning mechanisms that are needed to robustly acquire knowledge components of different complexity. For simple knowledge components, like the vocabulary associations discussed above, memory processes may be sufficient to acquire them. Somewhat more complex knowledge components, like learning French gender, require induction of a category structure. Rather than a simple association between constants, to learn gender requires matching nouns and modifiers in addition to learning to associate a given gender marker (le or la) across a variable set of items that make up the categories of masculine nouns and feminine nouns. But, because the acquired category structure must also be remembered, instructional methods that effectively engage memory processes are also required. This line of reasoning argues against the match hypothesis as it suggests that simpler memory enhancing instructional methods, like optimized scheduling, would also be effective for more complex knowledge components. What about more complex instructional methods for simpler knowledge components? To learn a single arbitrary association (or fact), a generalization process like that implemented by category induction, seems unneeded. Further, for an arbitrary association (e.g., that a Chinese character is pronounced "ching") an explanation structure to generate or derive a KC would not work. Re-derivation is only relevant for knowledge that has an underlying rationalization (complex knowledge) and not for arbitrary associations. These observations suggest the sufficient simplicity hypothesis is more nearly correct.

One counter argument to the sufficient simplicity hypothesis is that more complex instructional methods, like accountable talk or even self-explanation prompting, can indirectly achieve the same robust learning efficiency outcomes (like memory enhancement) that simpler methods (like optimized scheduling) achieve. Engaging basic memory processes may not be necessary to the extent that complex forms of instruction help students form an integrated network of knowledge (e.g., combinations of principles and reasoning strategies) that can be used to regenerate or re-derive forgotten knowledge. Supporters of mnemonic or memory elaboration strategies might take this argument further and suggest that robust learning of even the simplest paired associates can be enhanced by engaging learners in something like an explanation process (i.e., the mnemonic or elaboration is essentially an explanation).

To the extent the instructional goal is robust learning *efficiency*, we want learning outcomes that not only last and transfer, but that also are achieved without unnecessary time for students and instructions. Scientifically, we find that too many theoretical analyses and experimental studies do not address the time costs of instructional methods. Practically, use of more complex instructional strategies may not always be worth the extra time they tend to require.

We are not alone in suggesting that the generality of instructional principles may be bounded in predictable ways by the complexity of the targeted knowledge (cf., Sweller & Chandler, 1994; Wulf & Shea, 2002). We believe that a framework for addressing such issues in a broad and more systematic way is needed. The point of proposing the KLI theoretical framework effort is not to settle these issues. The goal is to provide a guide for generating new hypotheses and suggest new ways to probe the theoretical and empirical support for these hypotheses.

Conclusion

Our broad goal for this paper was to put forward a theoretical framework to organize the development of instructional theory at a grain size appropriate for guiding the design, development, and continual improvement of effective and efficient academic course materials, technologies, and instructor practices. This goal is consistent with purpose of the Pittsburgh Science of Learning Center, "to leverage cognitive theory and computational modeling to identify the conditions that cause robust student learning". It is also consistent with broader calls for cumulative theory development in education that is supported by "rigorous, sustained scientific research" (e.g., Shavelson & Towne, 2002). The need for cumulative theory is illustrated both by the general lack of consensus around educational practices that work and by the limitations of the large-scale randomized control trials as filters but not generators. In elaborating this framework, KLI, we described it's three fundamental taxonomies of kinds of knowledge, learning processes, and instructional methods, and outlined potential interdependencies between categories in these taxonomies, and illustrated how the framework can be used to generate new research questions.

In the developing the KLI framework, we emphasized the importance of knowledge components, as opposed to courses (Geometry; French) or topics (area, grammatical gender). Each domain and its many subdomains entail multiple knowledge components that are interrelated in various ways. Learning events are about which knowledge components are acquired in relation to which ones have already been acquired.

In contrast to Bloom's well-known taxonomy (Bloom, 1956), which is expressed in terms of instructional objectives, our taxonomy focuses on the knowledge needed to achieve those objectives and is expressed in a cognitive process representation. It is purposely at a more abstract and coarse-grained level than the representations used in computational models of cognition (e.g., Anderson & Lebiere, 1998; McClelland, Cleeremans, & Servan-Schreiber, 1990; Newell, 1990; Son, 1994). We conceive of knowledge as being decomposable into units that that relate some input characteristics or features of the student's perceived world or mental state to some output in the students changeable world or mental state. Unlike production rules in theories of cognitive architecture (Anderson & Lebiere, 1998; Newell, 1990), which are implicit components outside a students' awareness, the knowledge components in KLI include explicit, verbalizable knowledge. Given the prominence of comprehension, reasoning, dialogue, and argumentation in more complex forms of instruction (e.g., prompted self-explanation, accountable talk), the KLI knowledge taxonomy distinguishes between kinds of KCs that have rationales, such that students can effectively reason and argue about them, and some do not, such that such reasoning and argumentation may be of no or limited value for learning.

Learning occurs as unobservable events that can be inferred from performance and can be appropriately attributed to instruction events, under the circumstances of experimental control. The processes of learning include both simple associations and more complex, reflective processes that result in knowledge component changes of three broad types: 1) fluency building, 2) knowledge refinement, and 3) sense making. These learning processes can proceed more-or-less independently or in some synchrony.

Instructional principles emerge from research that is sufficiently convergent to support generalization. Instructional principles are intended to be widely applicable across domains and situations, although specific conditions of instructional environments can affect the implementation of the principle. Table 4 summarizes six such principles, including several that have received considerable PSLC research attention--feature focusing, prompted self-explanation, worked examples—and indeed have broad support from learning science research as a whole. In considering instructional principles, we confronted a challenge concerning the complexity of both instructional principles and knowledge components. We noted a general trend for correspondence between the complexity of the principle and that of the knowledge components to which it applies. It is tempting to summarize this relationship as "simple principles for simple knowledge, complex principles for complex knowledge". However, we think an alternative view could turn out to be more nearly correct: The complexity of the instructional principle should not exceed the complexity of the knowledge. However, these issues of alignment are far from clear at the moment.

One important entailment of our framework is the research questions it leads to. The KLI framework is not a set of frozen taxonomies but an interconnected set of theoretical and empirical claims that imply hypothesis-testing experiments. We developed the case of worked examples in enough detail to illustrate the richness of applying the framework to a single question that has been the focus of much research. Using the KLI framework and specifically the analysis of knowledge components led to experiments that found that instruction that was individualized on specific knowledge components led to more robust learning than instruction that was not (Salden et al, 2008). Such studies not only validate the basic assumption that knowledge components are the functional unit of analysis for learning, they suggest instructional procedures that can be the object of further research, leading at some point to a broad instructional principle.

As we discussed above, other researchers have recognized the importance for effective instructional design of a detailed analysis of domain content into the components of knowledge that students bring to a course and those we would like them to take away. Nevertheless, Anderson and Schunn (2000) expressed concern that "detailed cognitive analyses of various critical educational domains are largely ignored by psychologists, domain experts and educators". They noted the tendency for psychologists to value domain-general results, domain experts (e.g., mathematicians and linguists) to value new results in the domain, and educational researchers to value more holistic explanations. Some progress has been made (e.g., Clark, Feldon, van Merriënboer, Yates, & Early, 2007; Lee, 2003), but careful cognitive task analysis of domain knowledge is not a standard research practice in any discipline. Whether it is done through the emergence of a new discipline or through interdisciplinary teams, we hope to see such analysis become a more routine part of instructional design for new instructional domains as well as for existing ones.

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